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NATIONAL UNIVERSITY OF IRELAND, MAYNOOTH

THIRD COMPUTER SCIENCE AND SOFTWARE ENGINEERING EXAMINATION

SAMPLE 2003

PAPER SE307

COMPUTATION AND COMPLEXITY THEORY

Mr. T. Naughton.

Answer any THREE (3) questions. Time Allowed: 2 hours.

1. (a) Expand the languages defined by the following expressions. Note, e denotes the empty word, $\circ$ denotes concatenation, $\emptyset$ denotes the empty set, and 2^L denotes the power set of L. [6 marks]

   i. $\emptyset \cup \{aa, ab\}$
   ii. $\{e\}^*$
   iii. $\emptyset^*$
   iv. $\emptyset \circ \{a, b, c\}$
   v. 2^L, where the language $L = \{e, ab\}$
   vi. the regular expression $(0 \cup e)1$

   (b) Is it possible to enumerate the set of all words over a finite alphabet? Prove your answer. [6 marks]

   (c) For each of the following languages, prove that it is regular or prove that it is not regular. [13 marks]

   i. $\{w : w \in \{a, b\}^*, w$ is the empty word, or contains an odd number of as, or contains at least one $b\}$
   ii. $\{w : w \in \{a, b\}^*, w$ contains more as than bs$\}$
   iii. $\{uv : u, v \in \{a, b\}^*, u$ contains more as than $v\}$

2. (a) Prove that the regular languages are closed under union. [6 marks]

   (b) Explain the following properties of languages: *acceptable*, *decidable*, *recursively enumerable*, and *recursive*. Give an example in each case. [12 marks]

   (c) Prove that each of the following languages is a context-free language. [7 marks]

   i. $\{w : w \in \{a, b\}^*, w$ contains no more than two as$\}$
   ii. $\{w : w \in \{a, b\}^*, w$ contains more as than bs$\}$

3. (a) Construct a (deterministic or nondeterministic) Turing machine (TM) to accept the language L of 4-tuples (q, s, δ, t) where δ is the transition function of a nondeterministic finite automaton (NFA), where $q \in Q$ is the current state of the NFA, $s \in \Sigma$ is the input symbol, and $t \in Q$ is the next state according to δ. The four parts of the tuple will be written side-by-side on the input tape. The transition function δ will be written as a three symbol word with a # symbol preceeding each entry, and a # at the end of the last entry. The TM tape head will be positioned at the beginning of the input initially. The TM should write a 'T' at the end of the input and halt if the word is in the language. It does not matter what the TM does if the input word is not in the language, or if no entry of δ is suitable, or if the input word is badly formatted, as long as it does not write a 'T'. Assume that $Q = \{A, B\}$ and $\Sigma = \{0, 1\}$. Assume that δ contains exactly two entries. Indicate which is the initial state of your TM. As examples, "$A0\#A0B\#B1A\#B$" (without quotes) and "$B0\#A0B\#B0B\#B$" are valid words in L, and "$A0\#A0B\#B1A\#A$" is not. [10 marks]

(b) What steps are required to prove that a problem A is $\mathcal{NP}$-complete? State any $\mathcal{NP}$-complete problem ($\mathcal{NP}$-complete problems are always given in their decision form). [6 marks]

(c) Use a reduction to prove the undecidability of the VARINEQUALITY problem. VARINEQUALITY is defined as follows. Given a computer program P that takes no input, and two integer variables A and B declared in P, will the value in B ever exceed the value in A during the execution of P? [9 marks]

4. (a) The complement of a regular language is regular. The complement of a nonregular language is nonregular. Therefore, the language $L = \{uv : u, v \in \{a, b\}^*, u \text{ is not equal to } v\}$ is nonregular. Argue in support of, or against, this argument. [10 marks]

(b) A language is countable if it is recursively enumerable. But what about the converse of this statement: can we say that a language is recursively enumerable if it is countable? Give your technical opinion and support it. [10 marks]

(c) Let $\Sigma = \{0, 1\}$. Construct a FA, embedded in a TM, that accepts the language defined by the regular expression $\emptyset \cup e \cup \Sigma \cup \Sigma\Sigma$. [5 marks]