OLLSCOIL NA hÉIREANN, MÁ NUAD

NATIONAL UNIVERSITY OF IRELAND, MAYNOOTH

THIRD COMPUTER SCIENCE AND SOFTWARE ENGINEERING EXAMINATION

FEBRUARY 2003

PAPER SE307

COMPUTATION AND COMPLEXITY THEORY

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Answer any THREE (3) questions. Time Allowed: 2 hours.

1. (a) Expand the languages defined by the following expressions. Note, e denotes the empty word, $\circ$ denotes concatenation, $\emptyset$ denotes the empty set, and 2^L denotes the power set of L. [7 marks]

   i. $\emptyset \cup \{aa, ab\}$
   ii. $\{e\}^*$
   iii. $\emptyset^*$
   iv. $\emptyset \circ \{a, b, c\}$
   v. 2^L, where the language $L = \{e, ab\}$
   vi. the regular expression $(0 \cup e)1$
   vii. the context-free grammar $S \to SSS|SS|e$

   (b) Is it possible to enumerate the set of all words over a finite alphabet? Prove your answer. [5 marks]

   (c) For each of the following languages, prove that it is regular or prove that it is not regular. [13 marks]

   i. $\{w : w \in \{a, b\}^*, w$ is the empty word, or begins with a, or contains the substring $aab\}$
   ii. $\{wxw^R : w \in \{a, b\}^*\}$
   iii. $\{uv : u, v \in \{a, b\}^*, u$ is longer than $v\}$

2. (a) Prove that the regular languages are closed under the Kleene star operation. [6 marks]

   (b) Explain the following properties of languages: *acceptable*, *decidable*, *recursively enumerable*, and *recursive*. Give an example in each case. [8 marks]

   (c) Prove that each of the following languages is a context-free language. [11 marks]

   i. $\{w : w \in \{a, b\}^*, w$ contains exactly two as$\}$
   ii. $\{a^m b^n : m, n \geq 0, m = 2n\}$
   iii. $\{w : w \in \{a, b\}^*, w = w^R\}$

3. (a) Construct a (deterministic or nondeterministic) Turing machine (TM) to accept the language of correctly formatted transition functions for a nondeterministic finite automaton where $Q = \{A, B\}$, $\Sigma = \{0, 1\}$, $q_0 = A$, and $F = \{A\}$. A transition function δ will be written on the input tape as a single word with # symbols separating each entry of δ. Assume that δ will not contain multiple identical entries. Assume that the tape head is positioned at the beginning of the input initially. It is permissible for your TM to crash if it is presented with a badly formatted input. Your TM should write 'T' and halt if the input is a valid δ. Indicate which is the initial state of your TM. As examples, "$A0A\#A0B\#B0A$" (without quotes) and "" (the empty word) are valid possibilities for δ, and "$A01\#B0A$" and "$A0\#$" are not. [10 marks]

   (b) Problem C is known to be $\mathcal{NP}$-complete. A function f has been found that can transform instances of C into instances of problem V. What else is required to prove that V is $\mathcal{NP}$-complete? [4 marks]

   (c) Use a reduction to prove the undecidability of the VARHALTVAL problem. VARHALTVAL is defined as follows. Given a computer program P (that takes no input), an integer variable A declared in P, and an integer x, the solution to the problem is "yes" if P halts and when it halts A has the value x. Otherwise the solution to the problem is "no." [8 marks]

   (d) Define precisely the <u>language</u> equivalent to the VARHALTVAL problem. [3 marks]

4. (a) What does it mean for a language to be recursively enumerable but not recursive? Define two languages that are recursively enumerable but not recursive. [7 marks]

   (b) Prove or disprove each of the following. [12 marks]

      i. If a language is finite then a finite automaton can accept it.
      ii. If a language is countable then a finite automaton can accept it.
      iii. If a language is countable then a Turing machine can accept it.

   (c) What does a reduction $A \leq B$ between two problems A and B establish about the relative computability of A and B? What does a polynomial reduction establish about the relative computational complexity of A and B? [2 marks]

   (d) Given a program to enumerate all the permutations of a list of cities, what else is required to solve the decision form of the travelling salesman problem? [4 marks]