OLLSCOIL NA hÉIREANN, MÁ NUAD

NATIONAL UNIVERSITY OF IRELAND, MAYNOOTH

THIRD COMPUTER SCIENCE AND SOFTWARE ENGINEERING EXAMINATION

AUTUMN 2003

PAPER SE307

COMPUTATION AND COMPLEXITY THEORY

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Attempt any THREE questions. Time Allowed: 2 hours.

1. (a) Expand the languages defined by the following expressions. Where the language is infinite, list the first five words in the lexicographical ordering of the language. Note, e denotes the empty word, $\emptyset$ denotes the empty set, and $\Sigma = \{0, 1\}$. [7 marks]

   i. $(0 \cup 1)0^*$
   ii. Σ
   iii. $\Sigma^*\emptyset$
   iv. Σ^*1
   v. $0\Sigma^* \cup 1\Sigma^*$
   vi. $1 \cup e$
   vii. $1e$

   (b) The *type* of the value of an arithmetic expression (such as $3\times4+5$) is a number (17 in this case). What is the *type* of the value of a regular expression? What is the *type* of the value of a context-free grammar? [5 marks]

   (c) Is it possible to enumerate the set of all words over a finite alphabet? Prove your answer. [5 marks]

   (d) Prove that each of the following languages is a context-free language. [8 marks]

   i. $\{w : w \in \{a, b\}^*, w$ contains an even number of bs$\}$
   ii. $\{w : w \in \{a, b\}^*, w$ contains more as than bs$\}$

2. (a) Prove that the regular languages are closed under the concatenation operation. [6 marks]

   (b) Explain the following properties of languages: *acceptable*, *decidable*, and *recursively enumerable*. Give an example in each case. [6 marks]

   (c) For each of the following languages, prove that it is regular or prove that it is not regular. [13 marks]

      i. $\{w : w \in \{a,b\}^*, w$ has at least as many as as bs, w does not contain aa as a substring$\}$

      ii. $\{wxw^R : w \in \{a,b\}^*\}$

      iii. $\{uav : u,v \in \{a,b\}^*, u$ is longer than $v\}$

3. (a) Construct a (deterministic or nondeterministic) Turing machine (TM) to accept the language L of 4-tuples (q, s, δ, t) where δ is the transition function of a nondeterministic finite automaton (NFA), where $q \in Q$ is the current state of the NFA, $s \in \Sigma$ is the input symbol, and $t \in Q$ is the next state according to δ. The four parts of the tuple will be concatenated together and written on the input tape. The transition function δ will be written as a three symbol word with a $\#$ symbol preceding each entry, and a $\#$ at the end of the last entry. The TM tape head will be positioned at the beginning of the input initially. The TM should write a 'T' at the end of the input and halt if the word is in the language. It does not matter what the TM does if the input word is not in the language, or if no entry of δ is suitable, or if the input is badly formatted, as long as it does not write a 'T'. Assume that $Q = \{A, B\}$ and $\Sigma = \{0, 1\}$. Assume that δ contains exactly two entries. Indicate which is the initial state of your TM. As examples, "$A0\#A0B\#B1A\#B$" (without quotes) and "$B0\#A0B\#B0B\#B$" are valid words in L, and "$A0\#A0B\#B1A\#A$" is not. [15 marks]

   (b) The complement of a regular language is regular. The complement of a nonregular language is nonregular. Therefore, it is claimed that the language $L = \{uv : u,v \in \{a,b\}^*, u$ is not equal to $v\}$ is nonregular. Argue in support of, or against, this claim. [10 marks]

4. (a) Use a reduction to prove the undecidability of the VARINEQUALITY problem. VARINEQUALITY is defined as follows. Given a computer program P that takes no input, and two integer variables A and B declared in P, will the value in B ever exceed the value in A during the execution of P? [9 marks]

   (b) Make two small modifications to the definition of VARINEQUALITY such that in each case, the modified problem remains undecidable. [4 marks]

   (c) Define precisely the <u>language</u> equivalent to the VARINEQUALITY problem. [4 marks]

   (d) What steps are required to prove that a problem A is $\mathcal{NP}$-complete? State any $\mathcal{NP}$-complete problem ($\mathcal{NP}$-complete problems are always given in their decision form). [8 marks]