OLLSCOIL NA hÉIREANN, MÁ NUAD

NATIONAL UNIVERSITY OF IRELAND, MAYNOOTH

THIRD COMPUTER SCIENCE AND SOFTWARE ENGINEERING EXAMINATION

SAMPLE 2001

PAPER SE307

COMPLEXITY THEORY

Dr. A. Mycroft, Mr. S. Brown, Mr. T. Naughton

Answer ALL QUESTIONS from Section A and any TWO questions from Section B. Negative marking will be applied for Section A (2 marks for a correct answer, -0.5 for incorrect, 0 for no attempt). Time Allowed: 2 hours.

SECTION A (30 marks)

1. Given an alphabet Σ, a language over Σ is a [2 marks]

   (a) superset of 2^{Σ}
   (b) subset of Σ^*
   (c) superset of Σ^*
   (d) proper subset of Σ_0^*
   (e) none of the above

2. Which of the following functions has the largest growth rate? [2 marks]

   (a) $n^{1/2}$
   (b) $n^{1/10}$
   (c) n^{100}
   (d) $2^{n/2}$
   (e) $2^{n!}$

3. When n doubles [2 marks]

   (a) $\log n$ increments
   (b) a linear function in n doubles
   (c) a quadratic function in n quadruples
   (d) a quadratic function in n squares
   (e) an exponential function in n squares

4. The 'strongest' of the following statements we can make about the summation $1 + 2 + 3 + ... + n$ is [2 marks]

   (a) $= O(n^2)$
   (b) $= \frac{1}{2}n^2 + O(n)$
   (c) $= O(n^2) + O(n)$
   (d) $= O(n^2) - n$
   (e) $= n^2 + O(n)$

5. The sum $1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{1}{3}n(n + \frac{1}{2})(n + 1)$ would also be [2 marks]

   (a) $= O(n^3)$
   (b) $= \frac{1}{3}n + O(n^3)$
   (c) $= O(n^4)$
   (d) all of the above
   (e) none of the above

6. We can reduce our number of unique tape symbols in a TM table of behaviour by [2 marks]

   (a) increasing the size of the set of symbols
   (b) writing more symbols on the tape
   (c) increasing the number of states of mind
   (d) a method requiring both (b) and (c)
   (e) either (b) or (c)

7. The minimum number of symbols we can reduce our set of symbols to without changing the functionality of any possible TM is [2 marks]

   (a) 0
   (b) 1
   (c) 2
   (d) greater than 2
   (e) no limit

8. Which of the following are not one of the 'unrestricted' models of computation? [2 marks]

   (a) TMs with only one tape
   (b) k-tape TMs with a finite set of symbols
   (c) k-tape TMs whose tapes are infinite in one direction only
   (d) RAMs with fixed sized registers
   (e) none of the above

9. Given a countable set $\mathbf{A}$ of subsets of a set $\mathbf{X}$, where each $a \in \mathbf{A} \Rightarrow a \subseteq \mathbf{X}$, it can be said that [2 marks]

   (a) $\mathbf{A} \neq 2^{\mathbf{X}}$
   (b) $\mathbf{A} = 2^{\mathbf{X}}$
   (c) (cardinality of $\mathbf{A}$)>(cardinality of $2^{\mathbf{X}}$)
   (d) if $\mathbf{X}$ is not finite then $\mathbf{A}$ is not uncountable
   (e) none of the above

10. Which of the following languages is not recursively enumerable? [2 marks]

   (a) $\{$a, b, c$\}$
   (b) the odd integers
   (c) the prime numbers
   (d) the halting Turing machines
   (e) none of the above

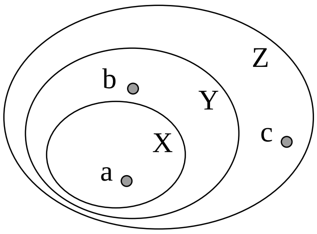
11. Which of the following languages is not recursive? [2 marks]

   (a) $\{$a, b, c$\}$
   (b) the odd integers
   (c) the prime numbers
   (d) the halting Turing machines
   (e) none of the above

12. The infinite set of all words over an alphabet Σ is denoted [2 marks]

   (a) Σ^*
   (b) Σ_0^*
   (c) $|\Sigma|$
   (d) 2^{Σ}
   (e) none of the above

13. Given the diagram below, depicting sets $\mathbf{X} \subseteq \mathbf{Y} \subseteq \mathbf{Z}$ and elements $a \in \mathbf{X}$, $b \in \mathbf{Y}$, $c \in \mathbf{Z}$, which of the following statements is false? [2 marks]



(a) b is $\mathbf{X}$-hard

(b) b is $\mathbf{Y}$-complete

(c) c is $\mathbf{Z}$-hard

(d) b is $\mathbf{Z}$-hard

(e) none of the above

14. Given a countable set $\mathbf{A}$ of subsets of a set $\mathbf{X}$, where each $a \in \mathbf{A} \Rightarrow a \subseteq \mathbf{X}$, it can be said that [2 marks]

(a) $\mathbf{A} \neq 2^{\mathbf{X}}$

(b) $\mathbf{A} = 2^{\mathbf{X}}$

(c) $|\mathbf{A}| > |2^{\mathbf{X}}|$

(d) if $\mathbf{X}$ is not finite then $\mathbf{A}$ is not uncountable

(e) none of the above

15. Given that a k-tape deterministic Turing machine T with $k \geq 1$ can be defined by the tuple $\langle Q, \Sigma, I, q_0, F \rangle$ which of the following is false? [2 marks]

(a) Σ always includes a 'blank' symbol

(b) I is a set of quintuples

(c) $q_0 \in Q$ is always the initial state

(d) $|F| \geq |Q|$

(e) none of the above

SECTION B (40 marks)

1. You are given a 2-tape Turing machine $T = \langle Q, \Sigma, I, q_0, F \rangle =$ $\langle \{00, 01, 02, 03, 09\}, \{0, 1, -\}, I, 00, \{09\} \rangle$ that operates on binary strings. As usual, the head of the first tape will be positioned at the beginning of the input and the second tape will be blank. I is

q	s	q'	s'	m
00	$\langle 0, - \rangle$	00	$\langle 0, - \rangle$	⟨R,S⟩
00	$\langle 1, - \rangle$	01	$\langle 1, - \rangle$	⟨R,S⟩
00	$\langle -, - \rangle$	02	$\langle -, - \rangle$	⟨L,S⟩
01	$\langle 0, - \rangle$	00	$\langle 0, - \rangle$	⟨R,S⟩
01	$\langle 1, - \rangle$	01	$\langle 1, - \rangle$	⟨R,S⟩
01	$\langle -, - \rangle$	03	$\langle -, - \rangle$	⟨L,S⟩
02	$\langle 0, - \rangle$	09	$\langle 0, 0 \rangle$	⟨S,S⟩
02	$\langle 1, - \rangle$	09	$\langle 1, 0 \rangle$	⟨S,S⟩
02	$\langle -, - \rangle$	09	$\langle -, - \rangle$	⟨R,S⟩
03	$\langle 0, - \rangle$	09	$\langle 0, 1 \rangle$	⟨S,S⟩
03	$\langle 1, - \rangle$	09	$\langle 1, 1 \rangle$	⟨S,S⟩
03	$\langle -, - \rangle$	09	$\langle -, - \rangle$	⟨R,S⟩

   (a) What does T do? Give as concise an explanation as you can. [7 marks]

   (b) Convert T into a functionally-identical Turing machine that requires at least one less state than T. [7 marks]

   (c) What would be the implications if a RAM algorithm solving an NP-complete problem was found to have (i) an exponential upper bound, or (ii) an exponential lower bound? [6 marks]

2. (a) Explain how we can view the construction of a deterministic Turing machine as a search through an ordered set. [4 marks]

   (b) Outline how a RAM (random-access machine) could be simulated on a Turing machine. Explain how your simulation lends weight to the Invariance thesis. [10 marks]

   (c) What does a polynomial reduction $A \leq B$ between two problems establish about their relative complexities? How could one use a reduction to prove non-membership of a class? [6 marks]

3. (a) Define the 1-D (one-dimensional) tiling problem and prove that it is decidable. [8 marks]

   (b) At a recent Clubs and Societies awards function a particularly bored individual got to thinking about whether it would be possible to nominate k clubs or societies for awards such that every participating student was a member of one, but only one, nominated club or society. Prove this problem is NP-complete. [12 marks]