SE120 - Discrete Structures II
Test 4
Friday 7 May 2004, 11:00, Lab4

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Instructions. Remove everything from your desk except pens/pencils. Paper will be provided. Answer all questions. Remember to be mathematically precise in all of your answers. You have until 11:45. You can leave as soon as you hand your script to an invigilator. The results for your previous tests will be available in the middle of next week.

1. Complete the following bijection between $\mathbb{N}$ and the set $B = \{b : b \in \mathbb{Z}, b \text{ is negative and odd}\}$: **[2 marks]**
   $f : \mathbb{N} \to B, f(x) =$ _____
   $g : B \to \mathbb{N}, g(x) =$ _____
2. Prove that the set $A = \{a : a \in \mathbb{N}$, when a is written in decimal it ends in a 7 (i.e. it has a 7 in its least significant position)$\}$ is countable. **[3 marks]**

Rules that can be applied in any question

Implication truth table:

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

Simplification (Simp): $\dfrac{A \wedge B}{A}$

Addition (Add): $\dfrac{A}{A \vee B}$

Conjunction (Conj): $\dfrac{A, B}{A \wedge B}$

Transitive: $\dfrac{a > b \wedge b > c}{a > c}$

Conditional Proof Rule (CP): If there is a proof of B from the assumption that A is true (i.e. if B can be derived from A), then $A \to B$

Assignment Axiom (AA): $\{Q(x/t)\}\, x := t\, \{Q\}$

Consequence Rule: $\dfrac{P \to R \quad \text{and} \quad \{R\}\, S\, \{Q\}}{\{P\}\, S\, \{Q\}}$

Composition Rule: $\dfrac{\{P\}\, S_1\, \{R\} \quad \text{and} \quad \{R\}\, S_2\, \{Q\}}{\{P\}\, S_1; S_2\, \{Q\}}$

If-Then Rule: $\dfrac{\{P \wedge C\}\, S\, \{Q\} \quad \text{and} \quad P \wedge \neg C \rightarrow Q}{\{P\}\ \textbf{if}\ C\ \textbf{then}\ S\, \{Q\}}$

If-Then-Else Rule: $\dfrac{\{P \wedge C\}\, S_1\, \{Q\} \quad \text{and} \quad \{P \wedge \neg C\}\, S_2\, \{Q\}}{\{P\}\ \textbf{if}\ C\ \textbf{then}\ S_1\ \textbf{else}\ S_2\, \{Q\}}$

While Rule: $\dfrac{\{P \wedge C\}\, S\, \{P\}}{\{P\}\ \textbf{while}\ C\ \textbf{do}\ S\, \{P \wedge \neg C\}}$

Statements that can be quoted without proof:

1. $\mathbb{N}$ is countable
2. Any set that has a bijection with a subset of $\mathbb{N}$ is countable
3. Let $B = A_1 \cup A_2 \cup \ldots \cup A_n$. If each A_i is countable then B is countable. If at least one A_i is uncountable then B is uncountable.
4. Let $B = A_1 \times A_2 \times \ldots \times A_n$. If each A_i is countable then B is countable. If at least one A_i is uncountable then B is uncountable.