SE120 - Discrete Structures II
Test 3
Thursday 22 April 2004, 17:00, Th1

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Instructions. Remove everything from your desk except pens/pencils. Paper will be provided. Answer all questions. Remember to be mathematically precise in all of your answers. You have until 17:45. You can leave as soon as you hand your script to an invigilator. There is no tutorial tomorrow.

1. Let $A = \{a, b, c\}, B = \{1, 2\}, C = \emptyset$. Write out the elements of each of the following relations by filling in the blanks. [**3 marks**]
   (a) $A \times B = \{$_____$\}$
   (b) $A \times B \times C = \{$_____$\}$
   (c) $R = \{$_____$\}$ is a relation over $A \times B$ and $|R| = 3$
2. Let R be a relation over $\mathbb{N} \times (\mathbb{Z} \times \mathbb{Z})$ defined as $R = \{(a, (b, c)) : a \in \mathbb{N}, b, c \in \mathbb{Z}, a = b + c\}$. [**4 marks**]
   (a) Explain why R is not a function.
   (b) Rewrite R (without changing its meaning) so that it becomes a function.
3. Let the problem WORDLENGTH be the problem of taking a word over the alphabet $\{a, b\}$ and calculating its length. Express this problem as a language acceptance problem. [**3 marks**]

Rules that can be applied in any question

Implication truth table:

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

Simplification (Simp): $\dfrac{A \wedge B}{A}$

Addition (Add): $\dfrac{A}{A \vee B}$

Conjunction (Conj): $\dfrac{A, B}{A \wedge B}$

Transitive: $\dfrac{a > b \wedge b > c}{a > c}$

Conditional Proof Rule (CP): If there is a proof of B from the assumption that A is true (i.e. if B can be derived from A), then $A \to B$

Assignment Axiom (AA): $\{Q(x/t)\}\, x := t\, \{Q\}$

Consequence Rule: $\dfrac{P \to R \quad \text{and} \quad \{R\}\, S\, \{Q\}}{\{P\}\, S\, \{Q\}}$

Composition Rule: $\dfrac{\{P\}\, S_1\, \{R\} \quad \text{and} \quad \{R\}\, S_2\, \{Q\}}{\{P\}\, S_1; S_2\, \{Q\}}$

If-Then Rule: $\dfrac{\{P \wedge C\}\, S\, \{Q\} \quad \text{and} \quad P \wedge \neg C \to Q}{\{P\}\ \textbf{if}\ C\ \textbf{then}\ S\, \{Q\}}$

If-Then-Else Rule: $\dfrac{\{P \wedge C\}\, S_1\, \{Q\} \quad \text{and} \quad \{P \wedge \neg C\}\, S_2\, \{Q\}}{\{P\}\ \textbf{if}\ C\ \textbf{then}\ S_1\ \textbf{else}\ S_2\, \{Q\}}$

While Rule: $\dfrac{\{P \wedge C\}\, S\, \{P\}}{\{P\}\ \textbf{while}\ C\ \textbf{do}\ S\, \{P \wedge \neg C\}}$

Statements that can be quoted without proof:

1. $\mathbb{N}$ is countable
2. Any set that has a bijection with a subset of $\mathbb{N}$ is countable
3. Let $B = A_1 \cup A_2 \cup \ldots \cup A_n$. If each A_i is countable then B is countable. If at least one A_i is uncountable then B is uncountable.
4. Let $B = A_1 \times A_2 \times \ldots \times A_n$. If each A_i is countable then B is countable. If at least one A_i is uncountable then B is uncountable.