

SE120 - Discrete Structures II
 Test 2 - Solutions
 Thursday 22 April 2004, 13:00, Th2

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1. (a) Prove $\{\text{true} \wedge x \geq 0\} x := x + 1 \{x > 0\}$ [4 marks]

1. $\{x + 1 > 0\} x := x + 1 \{x > 0\}$	AA
2. $\text{true} \wedge x \geq 0$	Proposition
3. $x \geq 0$	2,Simp
4. $x + 1 > x$	T
5. $x + 1 > x \wedge x \geq 0$	3,4,Conj
6. $x + 1 > 0$	5,Trans
7. $\text{true} \wedge x \geq 0 \rightarrow x + 1 > 0$	2,6,CP
8. $\{\text{true} \wedge x \geq 0\} x := x + 1 \{x > 0\}$	1,7,Consequence
- (b) Prove $\{\text{true} \wedge \neg(x \geq 0)\} x := -x \{x > 0\}$ [4 marks]

1. $\{-x > 0\} x := -x \{x > 0\}$	AA
2. $\text{true} \wedge \neg(x \geq 0)$	Proposition
3. $\neg(x \geq 0)$	2,Simp
4. $x < 0$	3,T
5. $-x > 0$	4,T
6. $\text{true} \wedge \neg(x \geq 0) \rightarrow -x > 0$	2,5,CP
7. $\{\text{true} \wedge \neg(x \geq 0)\} x := -x \{x > 0\}$	1,7,Consequence
2. (a) The loop invariant is $P = \{x \geq 0\}$. (This can be checked by showing that it is true immediately before and immediately after each iteration of the loop.) [4 marks]
- (b) Prove $P \wedge \neg C \rightarrow x = 0$ [4 marks]

1. $P \wedge \neg C$	P
2. $(x \geq 0) \wedge \neg(x > 0)$	1,T
3. $(x \geq 0) \wedge (x \leq 0)$	2,T
4. $x = 0$	3,T
5. $P \wedge \neg C \rightarrow x = 0$	1,4,CP
3. (a) 27 [1 mark]
- (b) No [1 mark]
- (c) No [1 mark]
- (d) No [1 mark]
- (e) No [1 mark]
- (f) No [1 mark]
- (g) (a, b) or any of the 9 elements of the set $A \times B = \{(a, b), (a, \{b\}), (a, \emptyset), (\{a\}, b), (\{a\}, \{b\}), (\{a\}, \emptyset), (\emptyset, b), (\emptyset, \{b\}), (\emptyset, \emptyset)\}$ [1 mark]
- (h) $\{(a, b), (\{a\}, \{b\})\}$ or $\emptyset$ or $\{(a, b)\}$ or any of the 512 elements of the set $2^{A \times B} = \{\emptyset, \{(a, b)\}, \{(a, \{b\})\}, \{(a, \emptyset)\}, \{(\{a\}, b)\}, \dots, \{(a, b), (a, \{b\})\}, \{(a, b), (a, \emptyset)\}, \dots\}$ [1 mark]