SE120 - Discrete Structures II
Test 2
Thursday 22 April 2004, 13:00, Th2

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Instructions. Remove everything from your desk except pens/pencils. Paper will be provided. Answer all questions. Remember to be mathematically precise in all of your answers. You have until 13:45. You can leave as soon as you hand your script to an invigilator.

1. Prove the correctness of the following computer program. [**8 marks**]
   $\{\text{true}\}$ **if** $x \geq 0$ **then** $x := x + 1$ **else** $x := -x$ $\{x > 0\}$

2. Find the loop invariant P for the following while loop and prove that $P \wedge \neg C \rightarrow x = 0$. [**8 marks**]
   $\{x \geq 99\}$
   while $x > 0$ **do**
   $\quad x := x - 1$
   od
   $\{x = 0\}$

3. Let $A = \{a, \{a\}, \emptyset\}, B = \{b, \{b\}, \emptyset\}$. Answer each of the following. [**8 marks**]
   (a) What is $|A \times A \times B|$? (I'm looking for an integer here.)
   (b) Is $\{\{a\}, b\} \in A \times B$?
   (c) Is $(\{b\}, a) \in A \times B$?
   (d) Is $[\{b\}, b] \in B \times B$?
   (e) Is $\{\emptyset\} \in A \times B$?
   (f) Is $\{\emptyset\} \subset A \times B$?
   (g) Give an element of $A \times B$ not already mentioned above.
   (h) Give an element of $2^{A \times B}$ not already mentioned above.

Rules that can be applied in any question

Implication truth table:

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

Simplification (Simp): $\dfrac{A \wedge B}{A}$

Addition (Add): $\dfrac{A}{A \vee B}$

Conjunction (Conj): $\dfrac{A, B}{A \wedge B}$

Transitive: $\dfrac{a > b \wedge b > c}{a > c}$

Conditional Proof Rule (CP): If there is a proof of B from the assumption that A is true (i.e. if B can be derived from A), then $A \to B$

Assignment Axiom (AA): $\{Q(x/t)\}\, x := t\, \{Q\}$

Consequence Rule: $$\frac{P \to R \quad \text{and} \quad \{R\}\, S\, \{Q\}}{\{P\}\, S\, \{Q\}}$$

Composition Rule: $$\frac{\{P\}\, S_1\, \{R\} \quad \text{and} \quad \{R\}\, S_2\, \{Q\}}{\{P\}\, S_1; S_2\, \{Q\}}$$

If-Then Rule: $$\frac{\{P \wedge C\}\, S\, \{Q\} \quad \text{and} \quad P \wedge \neg C \to Q}{\{P\}\ \textbf{if}\ C\ \textbf{then}\ S\, \{Q\}}$$

If-Then-Else Rule: $$\frac{\{P \wedge C\}\, S_1\, \{Q\} \quad \text{and} \quad \{P \wedge \neg C\}\, S_2\, \{Q\}}{\{P\}\ \textbf{if}\ C\ \textbf{then}\ S_1\ \textbf{else}\ S_2\, \{Q\}}$$

While Rule: $$\frac{\{P \wedge C\}\, S\, \{P\}}{\{P\}\ \textbf{while}\ C\ \textbf{do}\ S\, \{P \wedge \neg C\}}$$

Statements that can be quoted without proof:

1. $\mathbb{N}$ is countable
2. Any set that has a bijection with a subset of $\mathbb{N}$ is countable
3. Let $B = A_1 \cup A_2 \cup \ldots \cup A_n$. If each A_i is countable then B is countable. If at least one A_i is uncountable then B is uncountable.
4. Let $B = A_1 \times A_2 \times \ldots \times A_n$. If each A_i is countable then B is countable. If at least one A_i is uncountable then B is uncountable.