SE120 - Discrete Structures II
Test 1
Friday 5 March 2004, 11:00, Lab 4

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Instructions. Remove everything from your desk except pens/pencils. Paper will be provided for rough work. Use the MCQ sheet for your answers. Answer all questions. You have until 11:45. You can leave as soon as you hand your script to an invigilator.

1. Given the following Hoare triple and partial proof of correctness, what should go in the blank in line 1? Assume all variables are integer variables.

   $\{x > 5 \wedge y > 2\}\, x := y + 2\, \{x - 5 > 0\}$

   Proof.

1. $\{$___________$\}\, x := y + 4\, \{x - 5 > 0\}$	AA
2. ___________	proposition
3. $y > 2$	2,Simp
4. $y + 4 - 5 > 2 + 4 - 5$	3,T
5. $y + 4 - 5 > 1$	4,T
6. $1 > 0$	T
7. ___________	5,6,Transitive
8. $x > 5 \wedge y > 2 \rightarrow y + 4 - 5 > 0$	2,7,CP
QED	1,8,Conseq.

   (a) $x := y + 4$
   (b) $x - 5 > 0$
   (c) $x > 5 \wedge y > 2$
   (d) $x > 5$
   (e) $y + 4 - 5 > 0$

2. Given the proof in Question 1, what should go in the blank in line 2?

   (a) $x := y + 4$
   (b) $x - 5 > 0$
   (c) $x > 5 \wedge y > 2$
   (d) $x > 5$
   (e) $y + 4 - 5 > 0$

3. Given the proof in Question 1, what should go in the blank in line 7?

   (a) $y + 1 > y$
   (b) $y + 4 - 5 > 0$
   (c) $x > 5 \wedge y > 2$
   (d) $x := y + 4$
   (e) $x > 5$

4. What are the two subproofs required in the proof of correctness of the following triple? Assume all variables are integer variables.

   $\{\text{true}\}$ if $x \geq 0$ then $x := x + 1$ else $x := -x$ $\{x > 0\}$

   (a) Prove $\{\text{true} \wedge x \geq 0\}\, x := x + 1\, \{x > 0\}$ and $\{\text{true} \wedge \neg(x \geq 0)\}\, x := -x\, \{x > 0\}$
   (b) Prove $\{\text{true} \wedge x \geq 0\}\, x := -x\, \{x > 0\}$ and $\{\text{true} \wedge \neg(x \geq 0)\}\, x := x + 1\, \{x > 0\}$
   (c) Prove $\{\text{true}\}\, x := x + 1\, \{x := -x\}$ and $\{\text{false}\}\, x := -x\, \{x := x + 1\}$
   (d) Prove $\{\text{true}\}\, x := x + 1\, \{x > 0\}$ and $\{\text{false}\}\, x := -x\, \{x > 0\}$
   (e) Prove $\{\text{true} \wedge x \geq 0\}\, x := x + 1\, \{x \geq 0\}$ and $\{x \geq 0\}\, x := -x\, \{x > 0\}$

5. Given the following Hoare triple and partial proof of correctness, what should go in the blank in line 1? Assume all variables are integer variables.

   $\{x < 10\}$ if $x > 5$ then $x := 10 - x$ $\{x \leq 5\}$

   Proof.

1. $\{$____________$\}\, x := 10 - x\, \{x \leq 5\}$	AA
2. $x < 10 \wedge x > 5$	proposition
3. $x > 5$	2,Simp
4. ____________	3,T
5. $5 - x < 0$	4,T
6. $5 + 5 - x < 5$	5,T
7. $10 - x < 5$	6,T
8. $10 - x \leq 5$	7,Add
9. $x < 10 \wedge x > 5 \to 10 - x \leq 5$	2,8,CP
10. ____________	1,9,Conseq.
11. ____________	proposition
12. $\neg(x > 5)$	11,Simp
13. $x \leq 5$	12,T
14. $x < 10 \wedge \neg(x > 5) \to x \leq 5$	11,13,CP
QED	10,14,If-Then

   (a) $x \leq 5$
   (b) $x := 10 - x$
   (c) $x < 10$
   (d) $10 - x \leq 5$
   (e) None of the above

6. Given the proof in Question 5, what should go in the blank in line 4?

   (a) $-x \leq 5$
   (b) $-x \leq -5$
   (c) $x - x > 5 - x$
   (d) $10 > x > 5$
   (e) $x - 0 > 5 - 0$

7. Given the proof in Question 5, what should go in the blank in line 10?
   (a) $10 - x \leq 5$
   (b) $\neg(x > 5)$
   (c) $x < 10 \wedge x \leq 5$
   (d) $\{x < 10 \wedge x > 5\}\, x := 10 - x\, \{x \leq 5\}$
   (e) None of the above
8. Given the proof in Question 5, what should go in the blank in line 11?
   (a) $x < 10 \wedge \neg(x > 5)$
   (b) $x \geq 10 \wedge x < 5$
   (c) $x \leq 5$
   (d) $x \geq 10$
   (e) None of the above
9. Which expression can fill the blank in the following Hoare triple such that the triple evaluates to true. Assume all variables are integer variables.
   $\{x > 5 \wedge y > 5\}$ ____________ $\{x > 0 \wedge y > 5\}$
   (a) $x := -x$
   (b) $x := y$
   (c) $y := 0$
   (d) All of the above
   (e) None of the above
10. Which expression can fill the blank in the following Hoare triple such that the triple evaluates to true. Assume all variables are integer variables.
   $\{$____________$\}\, y := x + 2\, \{y > x \wedge x > 0\}$
   (a) $y > 0 \wedge x > 0$
   (b) true
   (c) $x + 2 > y$
   (d) $y + 2 > x$
   (e) None of the above

Rules

Implication truth table:

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

Simplification (Simp): $\dfrac{A \wedge B}{A}$

Addition (Add): $\dfrac{A}{A \vee B}$

Conjunction (Conj): $\dfrac{A, B}{A \wedge B}$

Transitive: $\dfrac{a > b \wedge b > c}{a > c}$

Conditional Proof Rule (CP): If there is a proof of B from the assumption that A is true (i.e. if B can be derived from A), then $A \to B$

Assignment Axiom (AA): $\{Q(x/t)\}\, x := t\, \{Q\}$

Consequence Rule: $\dfrac{P \to R \quad \text{and} \quad \{R\}\, S\, \{Q\}}{\{P\}\, S\, \{Q\}}$

If-Then Rule: $\dfrac{\{P \wedge C\}\, S\, \{Q\} \quad \text{and} \quad P \wedge \neg C \to Q}{\{P\}\ \textbf{if}\ C\ \textbf{then}\ S\, \{Q\}}$

If-Then-Else Rule: $\dfrac{\{P \wedge C\}\, S_1\, \{Q\} \quad \text{and} \quad \{P \wedge \neg C\}\, S_2\, \{Q\}}{\{P\}\ \textbf{if}\ C\ \textbf{then}\ S_1\ \textbf{else}\ S_2\, \{Q\}}$