SE120 - Discrete Structures II
Test 1 - Solutions
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1. As a consequence of $\{P\}S\{R\}$ evaluating to true, we know that if predicate P evaluates to true, and S is executed, then R will be true after S terminates.

2. From the If-Then-Else Rule we have to prove (i) $\{P \wedge C\}\, S_1\, \{Q\}$ and (ii) $\{P \wedge \neg C\}\, S_2\, \{Q\}$.

   (i) Prove $\{\text{true} \wedge \text{odd}(x)\}\, y := x + 2\, \{\text{odd}(y) \wedge y > x\}$

1. $\{\text{odd}(x+2) \wedge x+2 > x\}\, y := x + 2\, \{\text{odd}(y) \wedge y > x\}$	AA
2. $\text{true} \wedge \text{odd}(x)$	precondition
3. $\text{odd}(x)$	2,AND-simplification
4. $\text{odd}(x+2)$	3,T
5. $x+2 > x$	T
6. $\text{odd}(x+2) \wedge x+2 > x$	4,5
7. $\text{true} \wedge \text{odd}(x) \rightarrow \text{odd}(x+2) \wedge x+2 > x$	2,6
8. $\{\text{true} \wedge \text{odd}(x)\}\, y := x + 2\, \{\text{odd}(y) \wedge y > x\}$	1,7,consequence

   (ii) Prove $\{\text{true} \wedge \neg\text{odd}(x)\}\, y := x + 1\, \{\text{odd}(y) \wedge y > x\}$. Although not essential, let us first simplify this to $\{\text{even}(x)\}\, y := x + 1\, \{\text{odd}(y) \wedge y > x\}$

9. $\{\text{odd}(x+1) \wedge x+1 > x\}\, y := x + 1\, \{\text{odd}(y) \wedge y > x\}$	AA
10. $\text{even}(x)$	precondition
11. $\text{odd}(x+1)$	10,T
12. $x+1 > x$	T
13. $\text{odd}(x+1) \wedge x+1 > x$	11,12
14. $\text{even}(x) \rightarrow \text{odd}(x+1) \wedge x+1 > x$	10,13
15. $\{\text{even}(x)\}\, y := x + 1\, \{\text{odd}(y) \wedge y > x\}$	9,14,consequence
QED	8,15,If-Then-Else Rule

3. In order to calculate the loop invariant P, we must first reformat the while loop according to the While Rule. This gives us
   $\{x > 0 \wedge z \geq x\}$
   $y := 0;$
   $\{P\}$
   while $(x + y) < z$ **do**
     $y := y + 1$
   od
   $\{P \wedge \neg(x + y < z)\}$
   $\{x + y = z\}$

   From this, we can see that the second last line must imply the last line:
   $P \wedge \neg(x + y < z) \rightarrow x + y = z$
   and this gives us a method of solving for P.

The expression can be simplified to
$P \wedge x + y \geq z \rightarrow x + y = z$

The weakest predicate P (the predicate for P that allows the most possibilities) is therefore $P = x + y \leq z$.