OLLSCOIL NA hÉIREANN, MÁ NUAD

NATIONAL UNIVERSITY OF IRELAND, MAYNOOTH

FIRST COMPUTER SCIENCE AND SOFTWARE ENGINEERING EXAMINATION

SAMPLE 2003

PAPER SE120

DISCRETE STRUCTURES II

Mr. T. Naughton.

Attempt any THREE questions. Time Allowed: 2 hours.

1. (a) What is the meaning of the Hoare triple $\{P\}S\{R\}$ if it evaluates to true? [3 marks]

   (b) Prove that the following program is correct. [12 marks]
   {true}
   if odd(x) **then**
   $y := x + 2$
   else
   $y := x + 1$
   fi
   $\{\text{odd}(y) \wedge y > x\}$

   (c) Let a function $f : \mathbb{N} \to \mathbb{N}$ be defined as $f(x) = x \bmod 4$. Write an expression for the equivalence classes in the partition of $\mathbb{N}$ induced by the kernel relation of f. [10 marks]

2. (a) Define the language acceptance problem that corresponds to the problem of taking a list of integers and returning the list sorted in ascending order. [3 marks]

   (b) Given the binary relation $R = \{(1, 2), (2, 3), (3, 4), (4, 5)\}$, construct the relation R^3. [3 marks]

   (c) Prove that the set of all problems that one may wish a computer to solve is an uncountable set. Use a diagonalisation argument in your proof. [19 marks]

3. (a) Why are language acceptance (language recognition) problems of interest to computer scientists? [5 marks]

(b) Calculate the loop invariant P for the following program. [10 marks]

$\{x > 0 \land z \geq x\}$
$y := 0;$
$\{P\}$
while $(x + y) < z$ **do**
  $y := y + 1$
od
$\{x + y = z\}$

(c) For each of the following sets, state whether the set is finite, countably infinite, or uncountable. [4 marks]

i. The set of all real numbers less than 10.
ii. The set of all finite words over a finite alphabet.
iii. The set of all numbers divisible by π.
iv. The set of all people who are alive or have ever lived.

(d) Let R be the binary relation $R = \{(1,1),(2,2),(2,3),(3,2),(4,5)\}$ over $\{1,2,3,4,5\}$. R is not an equivalence relation. Transform R, with as little modification as possible, so that it becomes an equivalence relation. (Hint: modify R so that it is reflexive, symmetric, and transitive.) [6 marks]

4. (a) Let the set X be defined as $X = \{x|x \in \mathbb{R} \land 100x \in \mathbb{N}\}$. For example, $5.21 \in X$ and $0.99 \in X$. Prove that X is countable. [8 marks]

(b) Prove the correctness or incorrectness of each of the following computer programs. You must use the technique based on calculating the most general (or weakest) precondition. [12 marks]

i. $\{x > 0\}\, x := x * x; x := x \div 2\, \{x^4 = 10\}$
ii. $\{\text{true}\}\, x := 5; x := x + 1\, \{x > 5\}$

(c) Let $\sim$ be a relation on the natural numbers defined by $x \sim y$ iff mod$(x, 10) =$ mod$(y, 10)$. Use this relation to partition $\mathbb{N}$. [5 marks]

SE120 Axioms and Theorems

Implication truth table:

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

Assignment axiom (AA): $\{Q(x/t)\}\, x := t\, \{Q\}$

Consequence Rule: $$\frac{P \to R \quad \text{and} \quad \{R\}\, S\, \{Q\}}{\{P\}\, S\, \{Q\}}$$

Composition Rule: $$\frac{\{P\}\, S_1\, \{R\} \quad \text{and} \quad \{R\}\, S_2\, \{Q\}}{\{P\}\, S_1; S_2\, \{Q\}}$$

If-Then Rule: $$\frac{\{P \wedge C\}\, S\, \{Q\} \quad \text{and} \quad P \wedge \neg C \to Q}{\{P\}\ \textbf{if}\ C\ \textbf{then}\ S\, \{Q\}}$$

If-Then-Else Rule: $$\frac{\{P \wedge C\}\, S_1\, \{Q\} \quad \text{and} \quad \{P \wedge \neg C\}\, S_2\, \{Q\}}{\{P\}\ \textbf{if}\ C\ \textbf{then}\ S_1\ \textbf{else}\ S_2\, \{Q\}}$$

While Rule: $$\frac{\{P \wedge C\}\, S\, \{P\}}{\{P\}\ \textbf{while}\ C\ \textbf{do}\ S\, \{P \wedge \neg C\}}$$

Selected theorems that can be quoted without proof:
1. The union of any finite number of countable sets is a countable set
2. The cross product of any finite number of countable sets is a countable set
3. The intersection of any finite number of countable sets is a countable set
4. The power set of a finite set is a finite set