OLLSCOIL NA hÉIREANN, MÁ NUAD

NATIONAL UNIVERSITY OF IRELAND, MAYNOOTH

FIRST COMPUTER SCIENCE AND SOFTWARE ENGINEERING EXAMINATION

SUMMER 2003

PAPER SE120

DISCRETE STRUCTURES II

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Attempt any THREE questions. Time Allowed: 2 hours.

1. (a) What does it mean if a Hoare triple evaluates to true, and how can this be used to prove that a computer program is correct? Make reference to each of the three parts of the triple in your answer. [5 marks]

   (b) Prove the correctness or incorrectness of each of the following computer programs. You must use the technique based on calculating the most general (or weakest) precondition. [20 marks]

   i. $\{y < 0\}\, x := 5; x := x + y\, \{x < 10\}$
   ii. $\{x > 0\}\, x := x * x\, \{x^4 = 10\}$
   iii. $\{x > 5\}\, x := 5; x := x + 1\, \{x > 5\}$

2. (a) Let a function $f : \mathcal{N} \rightarrow \mathcal{N}$ be defined as $f(x) = \text{floor}(\sqrt{x})$. [15 marks]

   i. Write an expression for the equivalence classes in the partition of $\mathbb{N}$ induced by the kernel relation of f.
   ii. Write out the first four equivalence classes.

   (b) Prove the correctness of the following computer program. [10 marks]
   $\{x > 0\}$ **if** $x > 5$ **then** $x := 5\, \{0 \leq x \leq 5\}$

3. (a) Why are language acceptance (language recognition) problems of interest to computer scientists? [5 marks]

   (b) Let the set X be defined as $X = \{x | x \in \mathbb{R} \wedge 100x \in \mathbb{N}\}$. For example, $5.21 \in X$ and $0.99 \in X$. Prove that X is countable. [5 marks]

   (c) Prove the correctness of the following computer program. [15 marks]

   $\{x > 0\}$
   $y := 17;$
   $\{x > 0 \wedge y \leq 17\}$ // P
   while $x > y$ **do**
   $\quad x := x - y$
   od
   $\{0 < x \leq 17\}$

4. (a) Given the binary relation $R = \{(1, 2), (3, 1), (3, 2), (2, 4)\}$ over $\{1, 2, 3, 4\}$, construct each of the following relations. [9 marks]

   i. $r(R)$ (reflexive closure)
   ii. $s(R)$ (symmetric closure)
   iii. $t(R)$ (transitive closure)

   (b) Given some finite alphabet Σ, the set of all words over Σ is countable and is denoted $L = \Sigma^*$. Prove that the set of all languages over Σ, denoted 2^L or power(L), is uncountable. [16 marks]

SE120 Axioms and Theorems

Implication truth table:

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

Assignment axiom (AA): $\{Q(x/t)\}\, x := t\, \{Q\}$

Consequence Rule: $$\frac{P \to R \quad \text{and} \quad \{R\}\, S\, \{Q\}}{\{P\}\, S\, \{Q\}}$$

Composition Rule: $$\frac{\{P\}\, S_1\, \{R\} \quad \text{and} \quad \{R\}\, S_2\, \{Q\}}{\{P\}\, S_1; S_2\, \{Q\}}$$

If-Then Rule: $$\frac{\{P \wedge C\}\, S\, \{Q\} \quad \text{and} \quad P \wedge \neg C \to Q}{\{P\}\, \textbf{if}\ C\ \textbf{then}\ S\, \{Q\}}$$

If-Then-Else Rule: $$\frac{\{P \wedge C\}\, S_1\, \{Q\} \quad \text{and} \quad \{P \wedge \neg C\}\, S_2\, \{Q\}}{\{P\}\, \textbf{if}\ C\ \textbf{then}\ S_1\ \textbf{else}\ S_2\, \{Q\}}$$

While Rule: $$\frac{\{P \wedge C\}\, S\, \{P\}}{\{P\}\, \textbf{while}\ C\ \textbf{do}\ S\, \{P \wedge \neg C\}}$$

Selected theorems that can be quoted without proof:

1. The union of any finite number of countable sets is a countable set
2. The cross product of any finite number of countable sets is a countable set
3. The intersection of any finite number of countable sets is a countable set
4. The power set of a finite set is a finite set