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NATIONAL UNIVERSITY OF IRELAND, MAYNOOTH

FIRST COMPUTER SCIENCE AND SOFTWARE ENGINEERING EXAMINATION

AUTUMN 2003

PAPER SE120

DISCRETE STRUCTURES II

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Attempt any THREE questions. Time Allowed: 2 hours.

1. (a) Replace X in each of the following Hoare triples such that the triple evaluates to true. [9 marks]
   i. $\{X\}\, x := 5\, \{y > 5\}$
   ii. $\{x > 5 \wedge y > 5\}\, X\, \{x > 0 \wedge y > 5\}$
   iii. $\{x > 0 \wedge y > 5\}\, y := x\, \{X\}$

   (b) Let $P = \{x > 5 \wedge y < 0\}$. State two predicates that are strictly more general (weaker) than P and two predicates that are strictly less general (stronger) than P. [8 marks]

   (c) Define the language acceptance (language recognition) problem that corresponds to each of the following problems. [8 marks]
   i. The problem of taking two natural numbers and returning their sum.
   ii. The problem of taking a list of integers and returning the maximum value in the list.

2. (a) Determine if the following relation over $\mathbb{N}$ is an equivalence relation, by establishing if it is reflexive, symmetric, and transitive. [6 marks]
   aRb iff $|a - b| \leq 2$

   (b) For a finite alphabet of your choice, prove that the set of all languages over that alphabet is an uncountable set. Use a diagonalisation argument in your proof. [19 marks]

3. (a) Let $\sim$ be a relation on the natural numbers defined by $x \sim y$ iff $\text{mod}(x, 10) = \text{mod}(y, 10)$. Use this relation to partition $\mathbb{N}$. [5 marks]

   (b) Prove the correctness of the following computer program. You must determine the loop invariant P yourself. [20 marks]
   $\{x \geq 99\}$
   while $x > 0$ **do**
   $x := x - 1$
   od
   $\{x = 0\}$

4. (a) Prove the correctness of the following computer program. [8 marks]
$\{\text{true}\}$ **if** $x \geq 0$ **then** $x := x + 1$ **else** $x := -x$ $\{x > 0\}$

(b) Prove that the set of all Java computer programs is countable. [9 marks]

(c) Given the binary relation $R = \{(1,2),(2,3),(3,4),(4,5)\}$, construct each of the following relations. [8 marks]

i. R^2

ii. R^3

iii. R^4

iv. R^5

SE120 Axioms and Theorems

Implication truth table:

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

Assignment axiom (AA): $\{Q(x/t)\}\, x := t\, \{Q\}$

Consequence Rule: $$\frac{P \to R \quad \text{and} \quad \{R\}\, S\, \{Q\}}{\{P\}\, S\, \{Q\}}$$

Composition Rule: $$\frac{\{P\}\, S_1\, \{R\} \quad \text{and} \quad \{R\}\, S_2\, \{Q\}}{\{P\}\, S_1; S_2\, \{Q\}}$$

If-Then Rule: $$\frac{\{P \wedge C\}\, S\, \{Q\} \quad \text{and} \quad P \wedge \neg C \to Q}{\{P\}\, \textbf{if } C \textbf{ then } S\, \{Q\}}$$

If-Then-Else Rule: $$\frac{\{P \wedge C\}\, S_1\, \{Q\} \quad \text{and} \quad \{P \wedge \neg C\}\, S_2\, \{Q\}}{\{P\}\, \textbf{if } C \textbf{ then } S_1 \textbf{ else } S_2\, \{Q\}}$$

While Rule: $$\frac{\{P \wedge C\}\, S\, \{P\}}{\{P\}\, \textbf{while } C \textbf{ do } S\, \{P \wedge \neg C\}}$$

Selected theorems that can be quoted without proof:
1. The union of any finite number of countable sets is a countable set
2. The cross product of any finite number of countable sets is a countable set
3. The intersection of any finite number of countable sets is a countable set
4. The power set of a finite set is a finite set