OLLSCOIL NA hÉIREANN, MÁ NUAD

NATIONAL UNIVERSITY OF IRELAND, MAYNOOTH

THIRD COMPUTER SCIENCE EXAMINATION

FOURTH COMPUTER SCIENCE EXAMINATION

THIRD COMPUTER SCIENCE AND SOFTWARE ENGINEERING EXAMINATION

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MASTERS OF COMPUTER SCIENCE (YEAR 1) EXAMINATION

MASTERS OF COMPUTER SCIENCE (YEAR 2) EXAMINATION

SAMPLE 2004

PAPER CS355/SE307/CS403

COMPUTATION AND COMPLEXITY

Mr. T. Naughton

Attempt any THREE questions. Time Allowed: 2 hours.

1. (a) Construct a TM that recognises the language $L = \{a^n b^{2n} : n \geq 0\}$. You must write out the table of behaviour explicitly. Writing pseudocode alone will result in only minimal marks. [8 marks]

   (b) How can you use a reduction to prove nonmembership of a class? [5 marks]

   (c) Prove that the language $W = \{\langle M, w\rangle : M$ is a TM and when M is run on input string w, M runs for at least $2^{|w|}$ timesteps$\}$ is decidable. [8 marks]

   (d) Prove that PRINTERPROBLEM is in $\mathcal{NP}$. [4 marks]

2. (a) What is a model of computation? [5 marks]

   (b) Let the language $\text{INFINITE}_{\text{TM}}$ be defined as $\text{INFINITE}_{\text{TM}} = \{\langle M\rangle : M$ is a TM and $L(M)$ is an infinite language$\}$. Prove that $\text{INFINITE}_{\text{TM}}$ is undecidable. You are given that AT_{TM} is undecidable. AT_{TM} is defined as $\text{AT}_{\text{TM}} = \{\langle M, w\rangle : M$ is a TM and w is a word and M accepts $w\}$. [20 marks]

   You may answer this question by assigning a name, mathematical construct, or piece of pseudocode to each of the numbered blanks in the proof template in Figure 1. Where blanks have the same number, this denotes their contents will be the same. Alternatively, you can choose to ignore the template and construct your own proof from scratch (such as a subroutine reduction).

Proof. We will use a mapping reduction to prove the reduction __1__. Assume that __2__ is decidable. The function f that maps instances of __3__ to instances of __4__ is performed by TM F given by the following pseudocode.

$F =$ "On input $\langle$__5__$\rangle$:
1. Construct the following M' given by the following pseudocode.
   $M' =$ "__6__"
2. Output $\langle$__7__$\rangle$"

Now, $\langle$__7__$\rangle$ is an element of __8__ iff $\langle$__5__$\rangle$ is an element of __9__. So using f and the assumption that __2__ is decidable, we can decide __10__. A contradiction. Therefore, __2__ is undecidable. (This also means that the complement of __2__ is undecidable; the complement of any undecidable language is itself undecidable.)

Figure 1: Proof template for questions 2b and 3a.

3. Let the language $\text{NEVEROVERFLOW}_{\text{J}}$ be defined as $\text{NEVEROVERFLOW}_{\text{J}} = \{\langle J, v\rangle : J$ is a Java program, v is an integer variable declared in J, and throughout the execution of J variable v does not overflow$\}$. (A variable overflows when it exceeds its maximum value. For example, if an 8-bit unsigned integer with value 255 is incremented it might take the value 0, or cause a crash, but either way we would say that the variable has overflowed.) You are given that A_{J} is undecidable. A_{J} is defined $\text{A}_{\text{J}} = \{\langle J, w\rangle : J$ is a Java function that accepts on input $w\}$.

   (a) Prove that $\text{NEVEROVERFLOW}_{\text{J}}$ is undecidable. You may answer this question by assigning a name, mathematical construct, or piece of pseudocode to each of the numbered blanks in the proof template in Figure 1. Where blanks have the same number, this denotes their contents will be the same. Alternatively, you can choose to ignore the template and construct your own proof from scratch (such as a subroutine reduction). [15 marks]

   (b) Prove that $\text{NEVEROVERFLOW}_{\text{J}}$ is Turing recognisable or prove that it is not Turing recognisable. [5 marks]

   (c) Give a definition of the language $\overline{\text{NEVEROVERFLOW}_{\text{J}}}$ (the complement of $\text{NEVEROVERFLOW}_{\text{J}}$). Prove that $\overline{\text{NEVEROVERFLOW}_{\text{J}}}$ is Turing recognisable or prove that it is not Turing recognisable. [5 marks]

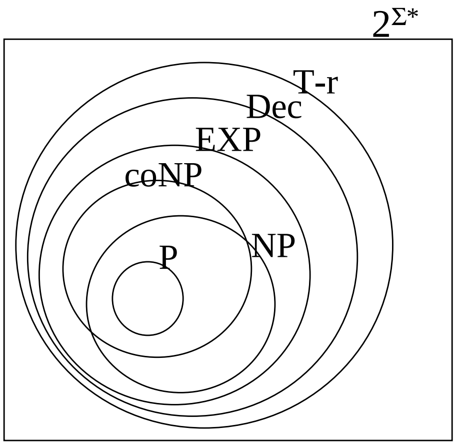

Figure 2: Illustration of the space of languages for question 4c: EXP denotes EXPTIME, Dec denotes the decidable languages (sometimes called the recursive languages), and T-r denotes the Turing recognisable languages (sometimes called the recursively enumerable languages).

4. (a) What is the Church-Turing thesis? [5 marks]

   (b) Prove that the set of all languages over the alphabet $\Sigma = \{a, b\}$ is an uncountable set. [10 marks]

   (c) Figure 2 illustrates the space of languages 2^{Σ^*} for some finite Σ, where $|\Sigma| > 1$. Place each of the following languages, <u>and</u> its complement, in its appropriate place in this space. [10 marks]

      i. $\mathrm{A}_{\mathrm{TM}} = \{\langle M, w\rangle : M$ is a TM and w is a word and M accepts $w\}$
      ii. $W = \{\langle M, w\rangle : M$ is a TM and when M is run on input string w, M runs for at least $2^{|w|}$ timesteps$\}$
      iii. PARTITION
      iv. $\mathrm{EQ}_{\mathrm{TM}} = \{\langle M_1, M_2\rangle : M_1, M_2$ are TMs and $L(M_1) = L(M_2)\}$
      v. $M = \{axbxc : a, b, c \in \{1\}^*, |c| = |a|.|b|\}$