NUI MAYNOOTH

Ollscoil na hÉireann Má Nuad

OLLSCOIL NA hÉIREANN, MÁ NUAD

THE NATIONAL UNIVERSITY OF IRELAND MAYNOOTH

Third Computer Science Examination

Third Computer Science and Software Engineering Examination

Masters Of Computer Science (Year 1)

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CS355/SE307/CS403
COMPUTATION AND COMPLEXITY

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Time allowed: 2 hours

Answer ***three*** questions

All questions carry equal marks

Proof. We will use a mapping reduction to prove the reduction __1__. Assume that __2__ is decidable. The function f that maps instances of __3__ to instances of __4__ is performed by TM F given by the following pseudocode.

$F =$ "On input $\langle$ __5__ $\rangle$:
1. Construct the following M' given by the following pseudocode.
   $M' =$ " __6__ "
2. Output $\langle$ __7__ $\rangle$"

Now, $\langle$ __7__ $\rangle$ is an element of __8__ iff $\langle$ __5__ $\rangle$ is an element of __9__. So using f and the assumption that __2__ is decidable, we can decide __10__. A contradiction. Therefore, __2__ is undecidable. (This also means that the complement of __2__ is undecidable; the complement of any undecidable language is itself undecidable.)

Figure 1: Proof template for questions 1a and 3b.

1. Let the language VARPOSV$_J$ be defined as VARPOSV$_J$ $= \{\langle J, v\rangle : J$ is a Java program, v is an integer variable declared in J, and when J is run the value in v never goes negative after v is initialised to a positive integer in its declaration statement$\}$. You are given that HALT$_J$ is undecidable. HALT$_J$ is defined HALT$_J$ $= \{\langle J, y\rangle : y \in \mathbb{Z}, J$ is a Java function that takes an integer argument and makes no function calls other than `System.out.Print()`, and J halts on input $y\}$.

   (a) Prove that VARPOSV$_J$ is undecidable. You may answer this question by assigning a name, mathematical construct, or piece of pseudocode to each of the numbered blanks in the proof template in Figure 1 on page 1. Where blanks have the same number, this denotes their contents will be the same. Alternatively, you can choose to ignore the template and construct your own proof from scratch (such as a subroutine reduction). [15 marks]

   (b) Prove that VARPOSV$_J$ is Turing recognisable or prove that it is not Turing recognisable. [5 marks]

   (c) Give a definition of the language $\overline{\text{VARPOSV}_J}$ (the complement of VARPOSV$_J$). Prove that $\overline{\text{VARPOSV}_J}$ is Turing recognisable or prove that it is not Turing recognisable. You may reuse your solutions to parts (b) and (c) in your proof. [5 marks]

2. (a) Prove that the set 2^S is uncountable where S is an arbitrary countable set. [10 marks]

   (b) Let PRINTERPROBLEM be defined as follows. You are required to print out a pile of n documents within a particular deadline. You have two printers at your disposal, each of which prints one page per second. Given a set $P = \{p_1, p_2, \ldots, p_n\}$ of nonnegative integers representing the number of pages in each of n documents to be printed, and a nonnegative integer t, can you print out all n documents in t seconds or less? You must print out whole documents at a time, but the order that the documents are printed is not important. Each instance of PRINTERPROBLEM has the form (n, P, t). Prove that PRINTERPROBLEM is $\mathcal{NP}$-complete. You are given that PARTITION $= \{A : A$ is a set of integers, and a subset of A exists called B such that the sum of the integers in B equals the sum of the integers in A but not in $B\}$ is $\mathcal{NP}$-complete. [15 marks]

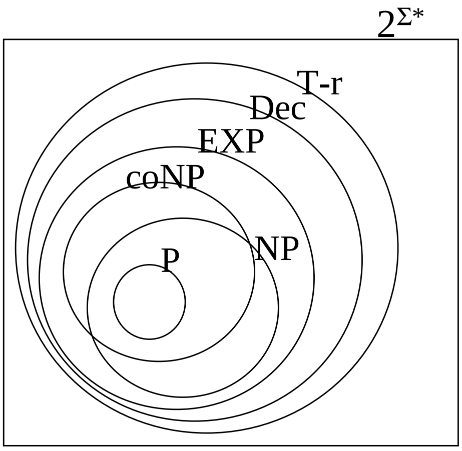

Figure 2: Illustration of the space of languages for question 4b: EXP denotes EXPTIME, Dec denotes the decidable languages (sometimes called the recursive languages), and T-r denotes the Turing-recognisable languages (sometimes called the recursively enumerable languages).

3. (a) What is a model of computation? [5 marks]

   (b) Let the language $\text{INFINITE}_{\text{TM}}$ be defined as $\text{INFINITE}_{\text{TM}} = \{\langle M \rangle : M$ is a TM and $L(M)$ is an infinite language$\}$. Prove that $\text{INFINITE}_{\text{TM}}$ is undecidable. You are given that AT_{TM} is undecidable. AT_{TM} is defined as $\text{AT}_{\text{TM}} = \{\langle M, w \rangle :$ M is a TM and w is a word and M accepts $w\}$. [20 marks]

   You may answer this question by assigning a name, mathematical construct, or piece of pseudocode to each of the numbered blanks in the proof template in Figure 1 on page 1. Where blanks have the same number, this denotes their contents will be the same. Alternatively, you can choose to ignore the template and construct your own proof from scratch (such as a subroutine reduction).

4. (a) How could you use a reduction to prove nonmembership of a class? [5 marks]

   (b) Figure 2 on page 2 illustrates the space of languages 2^{Σ^*} for some finite Σ, where $|\Sigma| > 1$. Place each of the following languages, **and** its complement, in its appropriate place in this space. [12 marks]

   i. $\text{HAL}_{\text{J}} = \{\langle J, v \rangle : J$ is a Java program and J and v is an integer variable declared in $J\}$

   ii. $\text{NRT}_{\text{J}} = \{\langle J_1, J_2 \rangle : J_1$ and J_2 are Java functions with identical return types$\}$

   iii. $\text{MEM}_{\text{J}} = \{\langle J_1, J_2 \rangle : J_1$ and J_2 are Java functions without arguments that require the same minimum amount of memory to run without crashing$\}$

   iv. $\text{AT}_{\text{TM}} = \{\langle M, w \rangle : M$ is a TM and w is a word and M accepts $w\}$

   v. $\text{NEM}_{\text{TM}} = \{\langle M \rangle : M$ is a TM and $L(M) \neq \emptyset\}$

   vi. $\text{NEQ}_{\text{J}} = \{\langle J_1, J_2 \rangle : J_1$ and J_2 are Java functions that recognise different languages$\}$

   (c) Construct a TM that recognises the language $L = \{a^n b^{2n} : n \geq 0\}$. You must write out the table of behaviour explicitly. Indicate the initial and accepting states. [8 marks]