CS403/SE307/CS355 - Computation and Complexity
Department of Computer Science
National University of Ireland, Maynooth

T Naughton, NUIM
http://www.cs.may.ie/~tnaughton/teaching/se307

Lab 4 - Decidability and undecidability - Group A - 18 Nov 2003 - Sample solutions

Question 1(i)
Proof that VAREQUAL_J is undecidable. We will use a mapping reduction to prove the reduction $\text{HALT}_\text{J} \leq \text{VAREQUAL}_\text{J}$. Assume that VAREQUAL_J is decidable. The function f that maps instances of HALT_J to instances of VAREQUAL_J is performed by TM F given by the following pseudocode.

$F =$ "On input $\langle J, w\rangle$:
1. Construct the following M' given by the following pseudocode.
   $M' =$ "class Mprime {
   public static void main(String args[]) {
   int $u = 5$;
   int $v = 6$;
   $J(w)$;
   $v{-}{-}$;
   }
   }"
2. Output $\langle M', u, v\rangle$"

Now, $\langle M', u, v\rangle$ is an element of VAREQUAL_J iff $\langle J, w\rangle$ is an element of HALT_J. So using f and the assumption that VAREQUAL_J is decidable, we can decide HALT_J. A contradiction. Therefore, VAREQUAL_J is undecidable.

Question 1(ii)
Proof that VAREQUAL_J is Turing recognisable. We construct a TM M to recognise VAREQUAL_J as follows.

$M =$ "On input $\langle J, u, v\rangle$:
1. If u and v are initialised to the same value in J, accept.
2. Run J.
3. After each line of J is executed, check if u and v are equal.
4. If they are equal, accept."

M recognises VAREQUAL_J. I.e. M is guaranteed to accept all instances of VAREQUAL_J and M will never accept triples $\langle J, u, v\rangle$ that are not instances of VAREQUAL_J. Therefore, VAREQUAL_J is Turing-recognisable.

Question 1(iii)
$\overline{\text{VAREQUAL}_\text{J}} = \{\langle J, u, v\rangle : J$ is a Java program, u and v are integer variables declared in J, and u and v never have the same value during the execution of $J\}$

Question 1(iv)
Proof that $\overline{\text{VAREQUAL}_\text{J}}$ is not Turing recognisable. VAREQUAL_J is undecidable and VAREQUAL_J is Turing recognisable, therefore $\overline{\text{VAREQUAL}_\text{J}}$ must not be Turing recognisable.