CS403/SE307/CS355 - Computation and Complexity Department of Computer Science National University of Ireland, Maynooth

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Lab 3 - Countable and uncountable sets - Group B - 4 Nov 2003 - Sample solutions

Question 1

Always. The cross-product of two countable sets A and B can be enumerated using a *breadth-first search*, as illustrated in the following diagram.



Using this technique, the elements of the cross-product would be enumerated in the following order: $(a_0, b_0), (a_0, b_1), (a_1, b_0), (a_0, b_2), (a_1, b_1), \ldots$

Question 2 Proof that $\mathbb{C}_{\mathbf{W}}$ is countable. The following TM *M* enumerates $\mathbb{C}_{\mathbf{W}}$.

M = "On input $\langle \rangle$:

1. Print out the whole complex numbers in the following order 0 + 0i, 0 + 1i, 1 + 0i, 0 - 1i, 1 + 1i, -1 + 0i, 0 + 2i, 1 - 1i, ... using a *breadth-first search* as shown below."



Therefore, \mathbb{C}_{W} is countable. (Note, it doesn't matter if multiple copies of the same element appear in our enumeration, as long as we don't leave any out.)