OLLSCOIL NA hÉIREANN, MÁ NUAD

NATIONAL UNIVERSITY OF IRELAND, MAYNOOTH

THIRD COMPUTER SCIENCE AND SOFTWARE ENGINEERING EXAMINATION

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PAPER SE307

COMPUTATION AND COMPLEXITY THEORY

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Answer ALL QUESTIONS from Section A and any TWO questions from Section B.

Use a MCQ Answer Sheet for Section A – enter your name, student ID, and module code. Negative marking will be applied for Section A (2 marks for a correct answer, -0.5 for an incorrect answer, 0 for no attempt).

Time Allowed: 2 hours.

SECTION A (40 marks)

1. Consider Turing machine T of the form (Q, Σ, I, q_0, F), where I is a set of tuples of the form (q, s, q', s', m), and where all symbols have their usual meaning. Which of the following conditions must be true for T to be a valid Turing machine? [2 marks]

   (a) T has exactly two tapes, even if one is not used

   (b) $F \subset Q$

   (c) Q is finite

   (d) $s \neq s'$ for each tuple in I

   (e) none of the above

2. Consider T from Question A.1. Which of the following conditions must be true for T to be a valid Turing machine? [2 marks]

   (a) if some Turing machine accepts a word w then T accepts w

   (b) each $i \in 2^{\Sigma}$ is a valid input

   (c) T halts on at least one input

   (d) all of the above

   (e) none of the above

3. Consider T from Question A.1. Which of the following conditions must be true for T to be a universal machine? [2 marks]

   (a) $q_0 \notin F$
   (b) $m \in \{\mathrm{L}, \mathrm{R}, \mathrm{S}\}$ for each tuple in I
   (c) $q \neq q'$ for each tuple in I
   (d) $F \neq Q$
   (e) none of the above

4. Consider T from Question A.1. Under which of the following restrictions will T definitely not be a universal machine? [2 marks]

   (a) $m \in \{\mathrm{L}, \mathrm{R}\}$ for each tuple in I
   (b) $m \in \{\mathrm{L}, \mathrm{S}\}$ for each tuple in I
   (c) Σ is finite
   (d) Q is finite
   (e) none of the above

5. Alice has a personal computer with 128 Mbytes of memory. Which of the following will increase the power (in terms of computability) of her computer? [2 marks]

   (a) adding more memory
   (b) increasing the clock speed of her processor
   (c) adding MMX (multimedia instructions) to her processor
   (d) all of the above
   (e) none of the above

6. Consider a finite alphabet A, and a finite word over A called w. Consider also a language over A called L. Which of the following is true for some A, w, and L? [2 marks]

   (a) $w \in L$
   (b) $w \in A$
   (c) $A \subseteq L$
   (d) all of the above
   (e) none of the above

7. Consider a finite alphabet A, and a finite word over A called w. Consider also a language over A called L. Which of the following is true for all A, w, and L? [2 marks]

   (a) $w \in L$
   (b) $w \in A$
   (c) $A \subseteq L$
   (d) all of the above
   (e) none of the above

8. The infinite set of all words over an alphabet Σ is denoted [2 marks]

   (a) Σ^*
   (b) Σ_0^*
   (c) $|\Sigma|$
   (d) 2^Σ
   (e) none of the above

9. Given a countably infinite set $\mathbf{A}$ of subsets of a set $\mathbf{X}$ (such that each $a \in \mathbf{A} \Rightarrow a \subseteq \mathbf{X}$) it can be said that [2 marks]

   (a) $\mathbf{A} \neq 2^{\mathbf{X}}$
   (b) $\mathbf{A} = 2^{\mathbf{X}}$
   (c) $|\mathbf{A}| > |2^{\mathbf{X}}|$, where $|\mathbf{A}|$ means 'the cardinality of $\mathbf{A}$'
   (d) $\mathbf{X}$ must be finite
   (e) none of the above

10. Which of the following is not one of the 'unrestricted' models of computation? [2 marks]

   (a) TMs with only one tape
   (b) k-tape TMs with a finite set of symbols
   (c) k-tape TMs whose tapes are infinite in one direction only
   (d) RAMs with a fixed number of registers
   (e) RAMs with fixed sized registers

11. Which of the following languages is not recursively enumerable? [2 marks]

   (a) $\{$a, b, c$\}$
   (b) the odd integers
   (c) the prime numbers
   (d) the halting Turing machines
   (e) none of the above

12. Which of the following languages is not recursive? [2 marks]

   (a) $\{$a, b, c$\}$
   (b) the odd integers
   (c) the prime numbers
   (d) the halting Turing machines
   (e) none of the above

13. Consider a 2-tape Turing machine $T = (Q, \Sigma, I, q_0, F) = (\{00, 01, 09\}, \{0, 1, -\}, I, 00, \{09\})$ that operates on binary strings. As usual, the head of the first tape will be positioned at the beginning of the input and the second tape will be blank. I is [2 marks]

q	s	q'	s'	m
00	$(0, -)$	00	$(0, 0)$	(R, R)
00	$(1, -)$	01	$(1, 1)$	(R, R)
00	$(-, -)$	09	$(-, -)$	(S, S)
01	$(0, -)$	00	$(0, 0)$	(R, R)
01	$(1, -)$	00	$(1, -)$	(R, S)
01	$(-, -)$	09	$(-, -)$	(S, S)

What would be written on the second tape when T halts on input “010110”?

(a) 01001

(b) 01010

(c) 01011

(d) T will not halt on that input

(e) none of the above

14. Consider T from Question A.13. What would be written on the second tape when T halts on input “11101”? [2 marks]

(a) 110

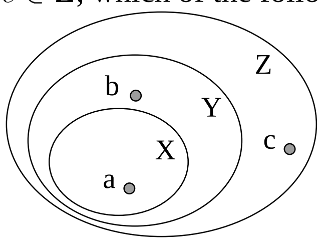
(b) 1101

(c) 101

(d) T will not halt on that input

(e) none of the above

15. Given the diagram below, depicting sets $\mathbf{X} \subseteq \mathbf{Y} \subseteq \mathbf{Z}$ and elements $a \in \mathbf{X}$, $b \in \mathbf{Y}$, $c \in \mathbf{Z}$, which of the following statements is false? [2 marks]



(a) a is $\mathbf{X}$-hard

(b) c is $\mathbf{X}$-hard

(c) b is $\mathbf{Y}$-complete

(d) c is $\mathbf{Z}$-hard

(e) none of the above

16. What would be the implications if an $\mathcal{NP}$-complete problem was found to have an exponential upper bound? [2 marks]

    (a) $\mathcal{P} = \mathcal{NP}$
    (b) $\mathcal{P} \neq \mathcal{NP}$
    (c) the RAM model would be thrown out of the class of reasonable machines (the first machine class)
    (d) it would lend weight to the Invariance thesis
    (e) there are no implications

17. What would be the implications if an $\mathcal{NP}$-complete problem was found to have an exponential lower bound? [2 marks]

    (a) $\mathcal{P} = \mathcal{NP}$
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18. What is required in order to prove that $\mathcal{NP}$-hard problem X is $\mathcal{NP}$-complete? Assume that problem Y is known to be $\mathcal{NP}$-complete. [2 marks]

    (a) find the polynomial reduction $Y \leq X$
    (b) show that a solution to X can be verified in polynomial time
    (c) show that the solution to an instance of X can be used to solve an instance of Y
    (d) all of the above
    (e) none of the above

19. You want to find an algorithmic solution for problem A. You know that A is in $\mathcal{NP}$. Should you look for an efficient algorithm for A? [2 marks]

    (a) yes, because we only suspect that $\mathcal{NP}$-complete problems are difficult
    (b) no, because the existence of $\mathcal{NP}$ has not been proved
    (c) yes, because $\mathcal{NP} \neq \mathcal{NP}$-complete
    (d) no, because the $\mathcal{NP}$-complete problems are outside $\mathcal{P}$
    (e) yes, because the $\mathcal{NP}$-complete problems are outside $\mathcal{NP}$

20. Is the problem of writing out the factorial of a number in unary $\mathcal{NP}$-complete or $\mathcal{NP}$-hard (e.g. $n! = 111111$ for $n = 3$)? [2 marks]

    (a) $\mathcal{NP}$-hard, because it cannot be solved efficiently
    (b) $\mathcal{NP}$-complete, because it cannot be solved efficiently
    (c) $\mathcal{NP}$-complete, because it can be verified, but not solved, in polynomial time
    (d) it is both $\mathcal{NP}$-complete and $\mathcal{NP}$-hard
    (e) it is neither $\mathcal{NP}$-complete nor $\mathcal{NP}$-hard

SECTION B (30 marks)

1. (a) Prove that the set of prime numbers is countable. [7 marks]

   (b) Explain the relationship between countable sets and computability. For example, why is it important for a language to be countable if it is to be accepted? [8 marks]

2. (a) Show how a reduction could be used to prove nonmembership of a class. [4 marks]

   (b) The European Space Agency (ESA) wants to avoid the software fault that caused the Arianne 5 rocket to fail. If they can ensure that variables will not overflow during program execution, then the rocket should not fail next time. They intend to check each function in the rocket's software separately. In order to ease the checking process, they have made the following simplifications to library functions: (i) function calls take only one argument, (ii) all variables are integers, (iii) there are no other function calls within a function, and (iv) the values of arguments are limited to a small finite range. Prove that, despite their simplifications, the problem of determining if a variable overflows in even one such function is unsolvable. [11 marks]

3. (a) "$\mathcal{NP}$ problems are unsolvable." Make three (3) concise points about the validity of the interpretation (or interpretations) of this statement. [6 marks]

   (b) "$\mathcal{NP}$-complete problems are the easiest of intractable problems." Do you agree with this statement? Explain your reasoning. [5 marks]

   (c) Define a language that is $\mathcal{P}$-hard. [4 marks]