
OLLSCOIL NA hÉIREANN, MÁ NUAD

NATIONAL UNIVERSITY OF IRELAND, MAYNOOTH

THIRD COMPUTER SCIENCE AND SOFTWARE ENGINEERING EXAMINATION

AUTUMN 2002

PAPER SE307

COMPUTATION AND COMPLEXITY THEORY

Dr. P. Stevens, Prof. R. Reilly, Mr. T. Naughton.

Answer ALL QUESTIONS from Section A and any TWO questions from Section B.

Use a MCQ Answer Sheet for Section A – enter your name, student ID, and module code. Negative marking will be applied for Section A (2 marks for a correct answer, -0.5 for an incorrect answer, 0 for no attempt).

Time Allowed: 2 hours.

SECTION A (40 marks)

1. Consider Turing machine T of the form (Q, Σ, I, q_0, F) , where I is a set of tuples [2 marks] of the form (q, s, q', s', m) , and where all symbols have their usual meaning. Which of the following conditions must be true for T to be a valid Turing machine?
 - (a) $F \subset Q$
 - (b) $Q \cap F \neq \{\}$
 - (c) $s \neq s'$ for at least one tuple in I
 - (d) $q_0 \notin F$
 - (e) none of the above
2. Consider T from Question A.1. Which of the following conditions must be true for [2 marks] T to be a valid Turing machine?
 - (a) if some Turing machine accepts a word w then T accepts w
 - (b) each $i \in 2^\Sigma$ is a valid input
 - (c) T halts on at least one input
 - (d) all of the above
 - (e) none of the above

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3. Consider T from Question A.1. Which of the following conditions must be true for T to be a universal machine? [2 marks]
- (a) $q_0 \notin F$
 - (b) $m \in \{L, R, S\}$ for each tuple in I
 - (c) $q \neq q'$ for each tuple in I
 - (d) $F \neq Q$
 - (e) none of the above
4. Consider T from Question A.1. Under which of the following restrictions will T definitely not be a universal machine? [2 marks]
- (a) $m \in \{L, R\}$ for each tuple in I
 - (b) $m \in \{L, S\}$ for each tuple in I
 - (c) Σ is finite
 - (d) Q is finite
 - (e) none of the above
5. Alice has a personal computer with 128 Mbytes of memory. Which of the following will increase the power (in terms of computability) of her computer? [2 marks]
- (a) adding more memory
 - (b) increasing the clock speed of her processor
 - (c) adding MMX (multimedia instructions) to her processor
 - (d) all of the above
 - (e) none of the above
6. Consider a finite alphabet A , and a finite word over A called w . Consider also a language over A called L . Which of the following is true for some A , w , and L ? [2 marks]
- (a) $w \in L$
 - (b) $w \in A$
 - (c) $A \subseteq L$
 - (d) all of the above
 - (e) none of the above
7. Consider a finite alphabet A , and a finite word over A called w . Consider also a language over A called L . Which of the following is true for all A , w , and L ? [2 marks]
- (a) $w \in L$
 - (b) $w \in A$
 - (c) $A \subseteq L$
 - (d) all of the above
 - (e) none of the above

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8. Given a finite alphabet Σ and a language L over Σ , it is always true that [2 marks]
- (a) L is finite
 - (b) L is infinite
 - (c) $L \neq \Sigma$
 - (d) $L = 2^\Sigma$
 - (e) none of the above
9. Given a finite alphabet Σ and a language L over Σ , it is always true that [2 marks]
- (a) L is empty
 - (b) $L \neq \Sigma$
 - (c) L is not countable
 - (d) 2^L is countable
 - (e) none of the above
10. Given a countably infinite set \mathbf{A} of subsets of a set \mathbf{X} (such that each $a \in \mathbf{A} \Rightarrow a \subseteq \mathbf{X}$) it can be said that [2 marks]
- (a) $\mathbf{A} \neq 2^{\mathbf{X}}$
 - (b) $\mathbf{A} = 2^{\mathbf{X}}$
 - (c) $|\mathbf{A}| > |2^{\mathbf{X}}|$, where $|\mathbf{A}|$ means ‘the cardinality of \mathbf{A} ’
 - (d) \mathbf{X} must be finite
 - (e) none of the above
11. Which of the following is not one of the ‘unrestricted’ models of computation? [2 marks]
- (a) TMs with exactly two tapes
 - (b) k -tape TMs where the following holds: $|\Sigma| < |Q|$
 - (c) k -tape TMs where the following holds: $|\Sigma| > 2 > |Q|$
 - (d) k -tape TMs with a two-dimensional tape, infinite in all four directions
 - (e) RAMs with fixed sized registers
12. Which of the following is not one of the ‘unrestricted’ models of computation? [2 marks]
- (a) TMs with only one tape
 - (b) k -tape TMs with a finite set of symbols
 - (c) k -tape TMs whose tapes are infinite in one direction only
 - (d) RAMs with fixed sized registers
 - (e) RAMs with a fixed number of registers

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13. Which of the following languages is not recursively enumerable? [2 marks]
- (a) the rationals
 - (b) the set of satisfiable conjunctive normal forms (SAT)
 - (c) the set of tiling kits that do not tile the plane
 - (d) the set of TMs that do not halt
 - (e) none of the above
14. Which of the following languages is not in \mathcal{NP} ? [2 marks]
- (a) the set of tuples (G, T, k) where G is a graph and T is a tour of that graph of length $\leq k$
 - (b) the set of satisfiable conjunctive normal forms (SAT)
 - (c) the set of tuples (a, b, c) where a is the product of prime numbers b and c
 - (d) the set of tuples (L, a) , where L is an unordered list of integers and a is the index of the largest value in L
 - (e) none of the above
15. What would be the implications if an \mathcal{NP} -complete problem was found to have an exponential upper bound? [2 marks]
- (a) $\mathcal{P} = \mathcal{NP}$
 - (b) $\mathcal{P} \neq \mathcal{NP}$
 - (c) the RAM model would be thrown out of the class of reasonable machines (the first machine class)
 - (d) it would lend weight to the Invariance thesis
 - (e) there are no implications
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17. What is required in order to prove that \mathcal{NP} -hard problem X is \mathcal{NP} -complete? [2 marks]
Assume that problem Y is known to be \mathcal{NP} -complete.
- (a) find the polynomial reduction $Y \leq X$
 - (b) show that a solution to X can be verified in polynomial time
 - (c) show that the solution to an instance of X can be used to solve an instance of Y
 - (d) all of the above
 - (e) none of the above
18. You want to find an algorithmic solution for problem A . You know that A is in \mathcal{NP} . [2 marks]
Should you look for an efficient algorithm for A ?
- (a) yes, because we only suspect that \mathcal{NP} -complete problems are difficult
 - (b) no, because the existence of \mathcal{NP} has not been proved
 - (c) yes, because $\mathcal{NP} \neq \mathcal{NP}$ -complete
 - (d) no, because the \mathcal{NP} -complete problems are outside \mathcal{P}
 - (e) yes, because the \mathcal{NP} -complete problems are outside \mathcal{NP}
19. Is the problem of writing out the factorial of a number in unary \mathcal{NP} -complete or \mathcal{NP} -hard (e.g. $n! = 111111$ for $n = 3$)? [2 marks]
- (a) \mathcal{NP} -hard, because it cannot be solved efficiently
 - (b) \mathcal{NP} -complete, because it cannot be solved efficiently
 - (c) \mathcal{NP} -complete, because it can be verified, but not solved, in polynomial time
 - (d) it is both \mathcal{NP} -complete and \mathcal{NP} -hard
 - (e) it is neither \mathcal{NP} -complete nor \mathcal{NP} -hard
20. Is matrix multiplication in \mathcal{NP} , or \mathcal{NP} -hard? [2 marks]
- (a) \mathcal{NP}
 - (b) \mathcal{NP} -hard
 - (c) both \mathcal{NP} and \mathcal{NP} -hard
 - (d) matrix multiplication is provably polynomial
 - (e) we do not have a provably optimal algorithm for matrix multiplication

SECTION B (30 marks)

1. (a) What impact does Gödel's Incompleteness theorem have on computer scientists today? [4 marks]
- (b) How did Gödel's technique of representing statements about numbers as numbers help Turing define his first uncomputable problem? [5 marks]
- (c) The playing of certain board games poses intractable problems for machines. One such game is called GOSLOWLY. It is not necessary to understand the rules of GOSLOWLY, but you are told that even the problem of deciding which player is the winner from the state of the board is \mathcal{NP} -hard. However this decision problem is *not* \mathcal{NP} -complete. Explain how this is possible, demonstrating an understanding of the underlined terms. [6 marks]

2. (a) Define a language L that is in the class of problems that are recursively enumerable but not recursive. You are given only that the language of programs/Turing machines that halt is in this class. [5 marks]
- (b) Sketch a proof that L is recursively enumerable. [5 marks]
- (c) Sketch a proof that L is not recursive. [5 marks]

3. (a) Define "computability" and "computational complexity". What is the difference? [4 marks]
- (b) What does a polynomial reduction $A \leq B$ between two problems establish about their relative complexities? How could we use a reduction to prove nonmembership of a class? [5 marks]
- (c) Is it true to say that finite languages are recursive and that infinite languages are not recursive? Prove that the finite language $\{aa, ab, bb, ba\}$ is recursive. [6 marks]