OLLSCOIL NA hÉIREANN MÁ NUAD

NATIONAL UNIVERSITY OF IRELAND, MAYNOOTH

THIRD COMPUTER SCIENCE AND SOFTWARE ENGINEERING EXAMINATION

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PAPER SE307

COMPLEXITY THEORY

Dr. A. Mycroft, Mr. S. Brown, Mr. T. Naughton.

Answer ALL QUESTIONS from Section A and any TWO questions from Section B.

Negative marking will be applied for Section A (2 for a correct answer, –0.5 for an incorrect answer, 0 for no attempt).

Time Allowed: 2 hours.

SECTION A (30 marks)

1 Measurement of something requires a comparison. What scale do we use for algorithms? [2 marks]

(a) sample inputs
(b) attributes such as worst/best/average performance
(c) ensuring a correct algorithm
(d) all of the above
(e) an infinite number of algorithms requires an infinite number of scales in the worst case

2 "In order to ensure that a machine would be able to answer every question from some formal system (say, arithmetic) we would have to permit it to give incorrect answers some of the time." This statement is [2 marks]

(a) provably true
(b) provably false
(c) a direct result of Church's thesis
(d) a direct result of Turing's thesis
(e) none of the above

3 "All machines have equal power (in terms of computability)." This statement is [2 marks]

(a) provably true
(b) provably false
(c) a direct result of Church's thesis
(d) a direct result of Turing's thesis
(e) none of the above

4 If at least one NP-complete problem was found to have polynomial complexity on a Turing machine then [2 marks]

(a) all NP problems would be in P
(b) at least one P problem would not be in NP
(c) the Invariance thesis would have a proof
(d) all of the above
(e) none of the above

5 Which of the following is necessary to describe the global state of a 1-tape deterministic Turing machine? [2 marks]

(a) the current line `executed' in the table of behaviour
(b) the current symbol being scanned
(c) the infinite tape
(d) the position of the tape head
(e) all of the above

6 Which of the following is not necessary to describe the global state of a 1-tape deterministic Turing machine? [2 marks]

(a) the finite number of symbols on the tape
(b) the current state
(c) the current symbol being scanned
(d) the position of the tape head
(e) all of the above are necessary

7 Any two infinite sets **A** and **B** are equinumerous if [2 marks]

(a) a bijection exists between one of them and the natural numbers
(b) a bijection exists between one of them and a proper subset of the other
(c) both are finite
(d) a bijection exists between them
(e) both sets can be enumerated or ordered

8 The `strongest' of the following statements we can make about the summation $1 + 2 + 3 + ... + n$ is [2 marks]

(a) $= O(n^2)$
(b) $= \frac{1}{2} n^2 + O(n)$
(c) $= O(n^2) + O(n)$
(d) $= O(n^2) - n$
(e) $= n^2 + O(n)$

9 Under what circumstances would it be preferable to use an algorithm with $(\log_2 n + 13)$ complexity rather than an algorithm with $(2^n + 9)$ complexity, where n is the input size? [2 marks]

(a) always
(b) never
(c) only when n is greater than 2
(d) only when n is greater than 3
(e) only when n is greater than 4

10 Given that a k-tape deterministic Turing machine T with $k \geq 1$ can be defined by the tuple $\langle Q, \Sigma, I, q_0, F \rangle$, which of the following is false? [2 marks]

(a) Q is always finite
(b) Q is a set of states
(c) Σ always is finite
(d) I always is finite
(e) none of the above

11 Given that a k-tape deterministic Turing machine T with $k \geq 1$ can be defined by the tuple $\langle Q, \Sigma, I, q_0, F \rangle$, which of the following is always true? [2 marks]

(a) $|F| \geq 2$
(b) if $|F|>2$ the TM is guaranteed to halt
(c) if $|F|<2$ the TM might halt
(d) if $|F|=1$ the TM is guaranteed to halt
(e) if $|F|=1$ the TM will never halt

12 Which of the following languages is not recursively enumerable? [2 marks]

(a) $\{a, b, c\}$
(b) the odd integers
(c) the prime numbers
(d) the halting Turing machines
(e) none of the above

13 Which of the following languages is not recursive? [2 marks]

(a) $\{a, b, c\}$
(b) the odd integers
(c) the prime numbers
(d) the halting Turing machines
(e) none of the above

14 The set of accepting Turing machines does not coincide with which of the following sets? [2 marks]

(a) the set of random access machines
(b) the set of recursive functions
(c) the set of partial recursive functions
(d) the set of recursively enumerable languages
(e) the set of C++ programs

15 The class NP denotes the set of all problems [2 marks]

(a) that provably do ***N***ot have ***P***olynomial solutions
(b) for which we have ***N***ot yet found ***P***olynomial solutions
(c) that have ***P***rovably ***N***ondeterministic solutions
(d) that have ***P***olynomial solutions for ***N***ondeterministic inputs
(e) none of the above

SECTION B (40 marks)

1 You are given the following Turing machine $\langle Q, \Sigma, I, q_0, F \rangle$ = $\langle \{00, 01, 02, 03, 09\}, \{1, -\}, I, 00, \{09\} \rangle$ that operates on unary numbers, where I is

q	***s***	***q'***	***s'***	***m***
00	1	00	1	R
00	–	01	1	R
01	1	01	1	R
01	–	02	–	L
02	1	03	–	L
02	–	09	–	S
03	1	03	1	L
03	–	09	–	S

where L, R, S, mean "move left one cell," "move right one cell," and "stay at current cell," respectively. Assume that the tape head is initially positioned at the beginning of the input.

(a) What does the Turing machine do? Where exactly is the tape head, relative to the input, at a halt? [6 marks]

(b) Are there any cases where the machine will not work? [2 marks]

(c) Are there any redundant rows in the table of behaviour? [4 marks]

(d) Show how you would convert this TM into a TM with identical functionality that only has tape head moves $m \in \{L, R\}$, i.e. it must move the tape head at each step. [4 marks]

(e) Given that what you have done in (d) can be done in the general case, what can you say about the relative computational power of TMs with $m \in \{L, R\}$ and TMs with $m \in \{L, R, S\}$. [4 marks]

2 (a) What was Turing's class of `computable numbers'? If there is an infinite number of numbers in this class, how can `uncomputable numbers' exist? [7 marks]

(b) State three undecidable properties of Turing machines. [5 marks]

(c) What can the complexity of a TM solution to a problem tell us about the complexity of a Java solution to that problem? What thesis supports your answer? Explain the thesis. [8 marks]

3 (a) How many times will a given partial recursive function f appear in the enumeration of partial recursive functions p_1, p_2, p_3, ... accepted by the set of TMs? [3 marks]

(b) Distinguish the terms 'hard' and 'complete' as used in computational complexity theory. Is it possible for a language that can be accepted with time complexity $O(2^n + n^3 + n \log_2 n)$ to be P-hard? [5 marks]

(c) In computer theory, how can accepting a language be considered equivalent to computing a function? [5 marks]

(d) What languages are in NP? Give an example of three NP languages with different complexities. [7 marks]