**OLLSCOIL NA hÉIREANN, MÁ NUAD**

**NATIONAL UNIVERSITY OF IRELAND, MAYNOOTH**

**B.SC. DEGREE (SINGLE AND DOUBLE HONOURS) EXAMINATION**

**MASTER OF COMPUTER SCIENCE EXAMINATION**

**SUMMER 2002**

**COMPUTER SCIENCE**

**PAPER CS403**

**COMPUTATIONAL COMPLEXITY THEORY**

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**Answer ALL QUESTIONS from Section A and any TWO questions from Section B.**

**Use a MCQ Answer Sheet for Section A – enter your name, student ID, and module code. Negative marking will be applied for Section A (2 marks for a correct answer, $-0.5$ for an incorrect answer, 0 for no attempt).**

**Time Allowed: 2 hours.**

## SECTION A (40 marks)

1. Consider Turing machine $T$ of the form $(Q, \Sigma, I, q_0, F)$, where $I$ is a set of tuples of the form $(q, s, q', s', m)$, and where all symbols have their usual meaning. Which of the following conditions must be true for $T$ to be a valid Turing machine? [2 marks]

   (a) if some Turing machine accepts a word $w$ then $T$ accepts $w$

   (b) each $i \in 2^{\Sigma}$ is a valid input

   (c) $T$ halts on at least one input

   (d) all of the above

   (e) none of the above

2. Consider $T$ from Question A.1. Which of the following conditions must be true for $T$ to be a universal machine? [2 marks]

   (a) $q_0 \notin F$

   (b) $m \in \{\text{L}, \text{R}, \text{S}\}$ for each tuple in $I$

   (c) $q \neq q'$ for each tuple in $I$

   (d) $F \neq Q$

   (e) none of the above

3. Consider $T$ from Question A.1. Under which of the following restrictions will $T$ definitely not be a universal machine? [2 marks]

   (a) $m \in \{\text{L}, \text{R}\}$ for each tuple in $I$
   (b) $m \in \{\text{L}, \text{S}\}$ for each tuple in $I$
   (c) $\Sigma$ is finite
   (d) $Q$ is finite
   (e) none of the above

4. Alice has a personal computer with $128$ Mbytes of memory. Which of the following will increase the power (in terms of computability) of her computer? [2 marks]

   (a) adding more memory
   (b) increasing the clock speed of her processor
   (c) adding MMX (multimedia instructions) to her processor
   (d) all of the above
   (e) none of the above

5. Consider a finite alphabet $A$, and a finite word over $A$ called $w$. Consider also a language over $A$ called $L$. Which of the following is true for all $A$, $w$, and $L$? [2 marks]

   (a) $w \in L$
   (b) $|L| < |A|$
   (c) $|w| > 0$
   (d) all of the above
   (e) none of the above

6. Consider a finite alphabet $A$, and a finite word over $A$ called $w$. Consider also a language over $A$ called $L$. Which of the following is true for some $A$, $w$, and $L$? [2 marks]

   (a) $2^L = A$
   (b) $w \in A$
   (c) $A \subset L$
   (d) all of the above
   (e) none of the above

7. Given a countably infinite set $\mathbf{A}$ of subsets of a set $\mathbf{X}$ (such that each $a \in \mathbf{A} \Rightarrow a \subseteq \mathbf{X}$) it can be said that [2 marks]

   (a) $\mathbf{A} \neq 2^{\mathbf{X}}$
   (b) $\mathbf{A} = 2^{\mathbf{X}}$
   (c) $|\mathbf{A}| > |2^{\mathbf{X}}|$, where $|\mathbf{A}|$ means 'the cardinality of $\mathbf{A}$'
   (d) $\mathbf{X}$ must be finite
   (e) none of the above

8. Which of the following is not one of the ‘unrestricted’ models of computation? [2 marks]

   (a) TMs with only one tape
   (b) $k$-tape TMs with a finite set of symbols
   (c) $k$-tape TMs whose tapes are infinite in one direction only
   (d) RAMs with a fixed number of registers
   (e) RAMs with fixed sized registers

9. What would be the implications if an $\mathcal{NP}$-complete problem was found to have an exponential upper bound? [2 marks]

   (a) $\mathcal{P} = \mathcal{NP}$
   (b) $\mathcal{P} \neq \mathcal{NP}$
   (c) the RAM model would be thrown out of the class of reasonable machines (the first machine class)
   (d) it would lend weight to the Invariance thesis
   (e) there are no implications

10. What would be the implications if an $\mathcal{NP}$-complete problem was found to have an exponential lower bound? [2 marks]

   (a) $\mathcal{P} = \mathcal{NP}$
   (b) $\mathcal{P} \neq \mathcal{NP}$
   (c) the RAM model would be thrown out of the class of reasonable machines (the first machine class)
   (d) it would lend weight to the Invariance thesis
   (e) there are no implications

11. What is required in order to prove that $\mathcal{NP}$-hard problem $X$ is $\mathcal{NP}$-complete? Assume that problem $Y$ is known to be $\mathcal{NP}$-complete. [2 marks]

   (a) find the polynomial reduction $Y \leq X$
   (b) show that a solution to $X$ can be verified in polynomial time
   (c) show that a solution to an instance of $X$ can be used to solve an instance of $Y$
   (d) all of the above
   (e) none of the above

12. How could one prove that problem $X$ is not a member of class $\mathcal{A}$? [2 marks]

   (a) find a reduction from $X$ to $Y$ where $Y \in \mathcal{A}$
   (b) find a reduction from $Y$ to $X$ where $Y \in \mathcal{A}$
   (c) find a reduction from $X$ to $Y$ where $Y \notin \mathcal{A}$
   (d) find a reduction from $Y$ to $X$ where $Y \notin \mathcal{A}$
   (e) none of the above

13. You want to find an algorithmic solution for problem $A$. You know that $A$ is in $\mathcal{NP}$. Should you look for an efficient algorithm for $A$? [2 marks]

   (a) yes, because we only suspect that $\mathcal{NP}$-complete problems are difficult
   (b) no, because the existence of $\mathcal{NP}$ has not been proved
   (c) yes, because $\mathcal{NP} \neq \mathcal{NP}$-complete
   (d) no, because the $\mathcal{NP}$-complete problems are outside $\mathcal{P}$
   (e) yes, because the $\mathcal{NP}$-complete problems are outside $\mathcal{NP}$

14. Why are solutions to $\mathcal{NP}$-complete problems difficult to find? [2 marks]

   (a) nobody has looked
   (b) the existence of $\mathcal{NP}$ has not been proved
   (c) the existence of problems complete for $\mathcal{NP}$ has not been proved
   (d) the solution to any one $\mathcal{NP}$-complete problem implies a solution to all others
   (e) solutions to $\mathcal{NP}$-problems are not difficult to find

15. The class $\mathcal{NP}$ contains [2 marks]

   (a) languages
   (b) solutions to problems
   (c) solutions to problems that can be verified in polynomial time
   (d) problems without polynomial solutions
   (e) none of the above

16. Is the problem of writing out the factorial of a number in unary $\mathcal{NP}$-complete or $\mathcal{NP}$-hard (e.g. $n! = 111111$ for $n = 3$)? [2 marks]

   (a) $\mathcal{NP}$-hard, because it cannot be solved efficiently
   (b) $\mathcal{NP}$-complete, because it cannot be solved efficiently
   (c) $\mathcal{NP}$-complete, because it can be verified, but not solved, in polynomial time
   (d) it is both $\mathcal{NP}$-complete and $\mathcal{NP}$-hard
   (e) it is neither $\mathcal{NP}$-complete nor $\mathcal{NP}$-hard

17. Is SAT in $\mathcal{NP}$, or $\mathcal{NP}$-hard? [2 marks]

   (a) $\mathcal{NP}$
   (b) $\mathcal{NP}$-hard
   (c) both $\mathcal{NP}$ and $\mathcal{NP}$-hard
   (d) it has not been proved that $\mathcal{NP}$ exists
   (e) 2-SAT is in $\mathcal{NP}$ while 3-(or more)SAT is in the class $\mathcal{NP}$-hard

18. Is matrix multiplication in $\mathcal{NP}$, or $\mathcal{NP}$-hard? [2 marks]

(a) $\mathcal{NP}$

(b) $\mathcal{NP}$-hard

(c) both $\mathcal{NP}$ and $\mathcal{NP}$-hard

(d) matrix multiplication is provably polynomial

(e) we do not have a provably optimal algorithm for matrix multiplication

19. Which of the following languages is not recursively enumerable? [2 marks]

(a) $\{i : i \in \mathbb{N}, i \bmod 3 = 0, i < 20\}$

(b) $\{w : w \in \{a,b\}^*,$ the number of $a$s and $b$s is the same$\}$

(c) the set of tiling kits that tile the plane

(d) $\{q : q \in \mathbb{Q}, r$ has a finite decimal expansion$\}$ (where $\mathbb{Q}$ is the set of rationals)

(e) none of the above

20. Which of the following languages is not recursive? [2 marks]

(a) $\{i : i \in \mathbb{N}, i \bmod 3 = 0, i > 20\}$

(b) $\{w : w \in \{a,b\}^*,$ the number of $a$s and $b$s is the same$\}$

(c) the set of tiling kits that tile the plane

(d) $\{q : q \in \mathbb{Q}, r$ has a finite decimal expansion$\}$ (where $\mathbb{Q}$ is the set of rationals)

(e) none of the above

## **SECTION B (30 marks)**

1. (a) State a valid interpretation of the Church-Turing thesis. [5 marks]

   (b) Find the reduction $A \leq B$, where $A$ is the problem of addition over $\mathbb{N}$ and $B$ is the travelling salesman problem. Prove your reduction is polynomial. [10 marks]

2. (a) What is the computational complexity equivalent of the Church-Turing thesis? State this thesis. [5 marks]

   (b) Define a language that is $\mathcal{P}$-hard. [4 marks]

   (c) "If $\mathcal{L}$ is not countable, then a Turing machine cannot accept $\mathcal{L}$." Prove this assertion. [6 marks]

3. (a) Explain in detail the steps required to prove that a problem $A$ is $\mathcal{NP}$-complete. [5 marks]

   (b) Arithmetic (addition, subtraction, multiplication, division, equality) over the integers is undecidable. Explain why this is so. How could you prove it? [10 marks]