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NATIONAL UNIVERSITY OF IRELAND, MAYNOOTH

B.SC. DEGREE (SINGLE AND DOUBLE HONOURS) EXAMINATION

MASTER OF COMPUTER SCIENCE EXAMINATION

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COMPUTER SCIENCE

PAPER CS403

COMPUTATIONAL COMPLEXITY THEORY

Dr. R. Procter, Mr. S. Brown, Mr. T. Naughton.

Answer ALL QUESTIONS from Section A and any TWO questions from Section B.

Use an MCQ Answer Sheet for Section A - enter your name, student id, and module code. Negative marking will be applied for Section A (2 for a correct answer, –0.5 for an incorrect answer, 0 for no attempt).

Time Allowed: 2 hours.

SECTION A (30 marks)

1 Given an algorithm, which of the following are required in order to determine its computational complexity? [2 marks]

(a) sample inputs and outputs
(b) assumptions about the underlying model of computation
(c) semantics of the algorithm
(d) all of the above
(e) none of the above

2 "In order to ensure that a machine would be able to answer every question from some formal system (say, arithmetic) we would have to permit it to give incorrect answers some of the time." This statement is [2 marks]

(a) provably true
(b) provably false
(c) a direct result of Church's thesis
(d) a direct result of Turing's thesis
(e) none of the above

3 The hypothesis that there is a notion of effective calculability independent of any particular formalism is [2 marks]

(a) provably true
(b) provably false
(c) a direct result of Church's thesis
(d) a direct result of Gödel's Incompleteness theorem
(e) a direct result of Turing's thesis

4 If every mathematical statement in a formal system can be either proved or disproved (i.e. proved false) then the system is [2 marks]

(a) consistent
(b) complete
(c) provably decidable
(d) complete or consistent but not both
(e) undefined because no sufficiently formal system can have such a property

5 If at least one NP-complete problem was found to have polynomial complexity on a Turing machine then [2 marks]

(a) all NP problems would be in P
(b) at least one P problem would not be in NP
(c) the Invariance thesis would have a proof
(d) all of the above
(e) none of the above

6 The set defined as “every finite thing in the universe except the word `cat', the letter `Q', and the number 5” is [2 marks]

(a) acceptable by a Turing machine
(b) decidable by a Turing machine
(c) recursively enumerable
(d) all of the above
(e) undecidable by a Turing machine

7 Under what circumstances would it be preferable to use an algorithm with $(2^n + 4)$ complexity rather than an algorithm with $(\log_2 n + 13)$ complexity, where n is the input size? [2 marks]

(a) only when n is less than 3
(b) only when n is less than 4
(c) when n is less than 5
(d) only when n is greater than 3
(e) none of the above

8 Given that a k-tape deterministic Turing machine T with $k \geq 1$ can be defined by the tuple $\langle Q, \Sigma, I, q_0, F \rangle$, which of the following is false? [2 marks]

(a) Σ always includes a `blank' symbol
(b) I is a set of quintuples
(c) $q_0 \in Q$ is always the initial state
(d) $|F| \geq |Q|$
(e) none of the above

9 Given that I is the set of quintuples of the form $\langle q, s, s', m, q' \rangle$ of a k-tape deterministic Turing machine T (with $k \geq 1$ and defined by the tuple $\langle Q, \Sigma, I, q_0, F \rangle$) which of the following is false? [2 marks]

(a) $s \in \Sigma^k$
(b) $q \neq q'$ for all quintuples
(c) $m \in \{L, R, S\}^k$
(d) no pair of quintuples has the same first two elements
(e) none of the above

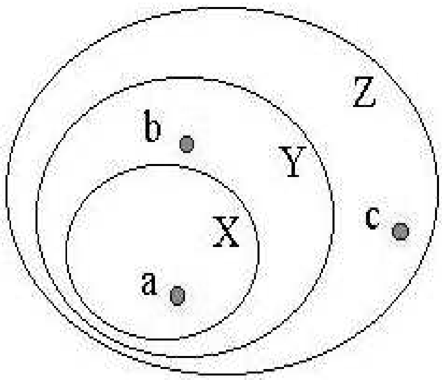
10 Which of the following models of computation is less powerful than the others? [2 marks]

(a) 1-tape Turing machines whose tapes are infinite in one direction only
(b) random access machines with finite numbers of registers
(c) lambda calculus
(d) partial recursive functions
(e) they all have equal power

11 The computational complexity class of languages PTIME, often referred to as P, [2 marks]

(a) denotes the set of languages decided by a Turing machine in polynomial time
(b) defines the set of tractably computable problems
(c) defines a lower bound on the complexity of Turing machines accepting these languages
(d) all of the above
(e) none of the above

12 Given the diagram below, depicting sets **X**⊆**Y**⊆**Z** and elements a∈ **X**, b∈ **Y**, c∈ **Z**, which of the following statements is false? [2 marks]



(a) a is X-hard
(b) b is X-complete
(c) c is Z-complete
(d) c is Y-hard
(e) none of the above

13 The class NP contains (among others) problems with algorithmic solutions that [2 marks]
(a) can be computed in polynomial time
(b) are provably intractable
(c) are polynomial for small inputs but not for larger inputs
(d) will never terminate
(e) cannot be classified with current mathematics

14 If a problem can be solved in polynomial time then [2 marks]
(a) the solution might still be intractable by requiring exponential space
(b) it can be verified in logarithmic time
(c) it can be verified in polynomial time
(d) it cannot be verified in all cases
(e) none of the above

15 If a solution to a problem cannot be verified in polynomial time then [2 marks]
(a) the solution cannot be verified in exponential time
(b) the verification process is not optimal
(c) the verification process is not decidable
(d) the problem is intractable
(e) none of the above since any solution can be verifiable in polynomial time

SECTION B (40 marks)

1 You are given Turing machine $T_1 = \langle Q, \Sigma, I, q_0, F\rangle$ = $\langle\{00, 01, 09\}, \{1, -\}, I, 00, \{09\}\rangle$ that operates on unary numbers, where I is

q	***s***	***q'***	***s'***	***m***
00	1	01	1	R
00	–	09	–	L
01	1	00	1	R
01	–	09	1	S

You are also given Turing machine $T_2 = \langle Q, \Sigma, I, q_0, F\rangle$ = $\langle\{00, 01, 02, 09\}, \{1, -\}, I, 00, \{09\}\rangle$ that operates on unary numbers, where I is

q	***s***	***q'***	***s'***	***m***
00	–	09	–	S
00	1	01	1	RR
01	1	01	1	RR
01	–	02	–	L
02	–	09	1	S
02	1	09	1	S

and where RR means move two (2) tape cells to the right in one step.

(a) What does each Turing machine do? Where exactly is each tape head, relative to the input, as each one halts? [9 marks]

(b) How many table of behaviour lookups are required for each of T_1 and T_2 as the length n of the input increases? [8 marks]

(c) In the general case, by how much more powerful (in terms of computability) is a machine that allows {RR, LL, R, L, S} tape head movements than a machine that only allows {R, L, S} movements? [3 marks]

2 (a) Explain how we can view the construction of a deterministic Turing machine as a search through an ordered set. [4 marks]

(b) What are the 'unrestricted models of computation'? What conjecture links them together? Name four such models, detailing one. [10 marks]

(c) State a valid interpretation of Church's thesis. State a valid interpretation of Turing's thesis. [6 marks]

3 (a) Define "computability" and "complexity". What is the difference? [4 marks]

(b) How are the properties *recursively enumerable*, *recursive*, and *partial recursive* related? [5 marks]

(c) What does a polynomial reduction $A \leq B$ between two problems establish about their relative complexities? How could we use a reduction to prove nonmembership of a class? [5 marks]

(d) The playing of certain board games pose <u>intractable</u> problems for machines. One such game is called GOSLOWLY. It is not necessary to understand the rules of GOSLOWLY, but you are told that even the problem of deciding which player is the winner from the state of the board is NP-<u>hard</u>. However this decision problem is *not* NP-<u>complete</u>. Explain how this is possible, demonstrating an understanding of the underlined terms. [6 marks]