OLLSCOIL NA hÉIREANN, MÁ NUAD

NATIONAL UNIVERSITY OF IRELAND, MAYNOOTH

B.SC. DEGREE (SINGLE AND DOUBLE HONOURS) EXAMINATION

MASTER OF COMPUTER SCIENCE EXAMINATION

SUMMER 2000

COMPUTER SCIENCE

PAPER CS403

COMPUTATIONAL COMPLEXITY THEORY

Dr. A. Mycroft, Mr. S. Brown, Mr. T. Naughton.

Attempt any THREE questions. Time Allowed: 2 hours.

1 (a) What is computational complexity theory and why is it useful to computer scientists? [5 marks]

(b) State the theorem relating simulations of *k*-tape TMs on 1-tape TMs. [5 marks]

(c) Verify your answer to (b) with Turing machines to recognise the language $a^n b a^n b^n$ = (b, abab, aabaabb, ...). For both the 1-tape and *k*-tape Turing machines you will have to [15 marks]
(*i*) produce a table of behaviour or state diagram,
(*ii*) calculate the exact cost in terms of number of table lookups (TIME) and tape cells used (SPACE), and
(*iii*) express these costs in asymptotic notation.

2 (a) What do we mean when we say that computational complexity theory relies on the Invariance thesis? [5 marks]

(b) What would be the implications if a RAM algorithm solving an NP-complete problem was found to have (*i*) an exponential upper bound, or (*ii*) an exponential lower bound? [6 marks]

(c) State three undecidable properties of Turing machines. [5 marks]

(d) What languages are in NP? Give an example of three NP languages with different complexities. [9 marks]

3 (a) Explain how we can view the construction of a deterministic Turing machine as a search through an ordered set. [4 marks]

(b) Outline how a RAM (random-access machine) could be simulated on a Turing machine. Explain how your simulation lends weight to the Invariance thesis. [10 marks]

(c) What would the worst case complexity of your simulation from (b) be if you used a 2-tape machine? And a 1-tape machine? [5 marks]

(d) What does a polynomial reduction $A \leq B$ between two problems establish about their relative complexities? How could one use a reduction to prove non-membership of a class? [6 marks]

4 (a) Define the 1-D (one-dimensional) tiling problem and prove that it is decidable. [8 marks]

(b) At a recent Clubs and Societies awards function a particularly bored individual got to thinking about whether it would be possible to nominate k clubs or societies for awards such that every participating student was a member of one, but only one, nominated club or society. Prove this problem is NP-complete. [11 marks]

(c) Is it true to say that finite languages are recursive and that infinite languages are not recursive? Prove that the finite language {aa, ab, bb, ba} is recursive. [6 marks]

5 (a) You want to find an algorithmic solution for problem A. You know that A is in NP. Should you look for an efficient algorithm for A? [3 marks]

(b) How are the properties recursively enumerable, recursive, and partial recursive related. [5 marks]

(c) 'Computability' is a concept with a precise mathematical definition. Turing presented a definition in 1936. What did it involve? You are not required to describe Turing's machine. [7 marks]

(d) Prove the unsolvability of the halting problem. [10 marks]

6 (a) If the time complexity of a k-tape deterministic Turing machine is $t(n)$, what is the upper bound on its space complexity $s(n)$? If the space complexity of a k-tape deterministic Turing machine is $s(n)$ how can we calculate an upper bound on its time complexity? [6 marks]

(b) What is the linear speedup theorem? Illustrate with an example. [5 marks]

(c) Is the problem of writing out the factorial of a number in unary NP-complete or NP-hard (e.g. $n! = 111111$ for $n = 3$)? Explain. [5 marks]

(d) Explain why DTMs can be regarded as a special case of NTMs? [5 marks]

(e) Give two different technical meanings for each of the terms *powerful* and *feasible*. [4 marks]