CS370 Lab Sheet 6, 13 Dec 2005

1. Let CONNECTED = {(G,a,b) : G=(V,E) is a graph, a,b are elements of V, and there is a path from a to b in G}. Prove that CONNEXTED is in NP.

2. Let SQUARE = {L : L is an array of integers and L contains two integers a and b such that $a^2 = b$}. Prove that SQUARE is in NP.

3. Let SUBSETSUM = {(S, k) : S is a set nonnegative integers, k is a nonnegative integer, and some subset of elements in S add up to the value k}. Prove that SUBSETSUM is in NP.

Definition. Partition: when we partition a set X we divide it into a collection of subsets such that the union of the subsets is X and no element of X appears in more than one subset.

4. Let BINPACKING2 = {A : A is a set of nonnegative integers and A can be partitioned into two subsets such that the sum of the integers in each subset is exactly the same}. Prove that BINPACKING2 is in NP.

5. Let BINPACKING = {(A, k) : A is a set of nonnegative integers, k is a nonnegative integer, and A can be partitioned into k subsets such that the sum of the integers in each subset is exactly the same}. Prove that BINPACKING is in NP.

Corrector's sample solution:

3. Let SUBSETSUM = {(S, k) : S is a set nonnegative integers, k is a nonnegative integer, and some subset of elements in S add up to the value k}. Prove that SUBSETSUM is in NP.

Proof: We will prove this by showing the existence of a TM M that verifies a solution to SUBSETSUM in polynomial time. The certificate will be the subset of S that adds up to k.

M = "On input (S, k, c):

1. Check that each element in c is in S.
2. Check that the sum of the elements in c is equal to k.
3. If both checks are OK, accept, else reject."

Step 1 takes n^2 steps where $n=|S|$.
Step 2 takes n steps.
Step 3 takes 1 step.

The total time that M takes in the worst case is n^2+n+1 which is a polynomial. Therefore this proves that SUBSETSUM is in NP.