OLLSCOIL NA hÉIREANN, MÁ NUAD

NATIONAL UNIVERSITY OF IRELAND, MAYNOOTH

B.SC. COMPUTER SCIENCE EXAMINATION

B.SC. COMPUTER SCIENCE AND SOFTWARE ENGINEERING EXAMINATION

MASTER OF COMPUTER SCIENCE EXAMINATION

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PAPER CS403

COMPUTATIONAL COMPLEXITY THEORY

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Attempt any THREE questions. Time Allowed: 2 hours.

1. (a) Expand the languages defined by the following expressions. Where the language is infinite, list the first five words in the lexicographical ordering of the language. Note, e denotes the empty word, $\emptyset$ denotes the empty set, and $\Sigma = \{0, 1\}$. [9 marks]

   i. $(0 \cup 1)0^*$
   ii. Σ
   iii. $\Sigma^*\emptyset$
   iv. Σ^*1
   v. $0\Sigma^* \cup 1\Sigma^*$
   vi. $1 \cup e$
   vii. $1e$
   viii. e^*
   ix. $\emptyset^*$

   (b) The *type* of the value of an arithmetic expression (such as $3\times4+5$) is a number (17 in this case). What is the *type* of the value of a regular expression? What is the *type* of the value of a context-free grammar? [5 marks]

   (c) Prove that each of the following languages is a context-free language. [11 marks]

   i. $\{vw : v \in \{a, b\}, w \in \{a, b\}^*, w$ contains twice as many as as bs if $v = a, w$ contains twice as many bs as as if $v = b$ $\}$
   ii. $\{w : w \in \{a, b\}^*, w$ contains no less than two as$\}$
   iii. $\{w : w \in \{a, b\}^*, w$ contains more as than bs$\}$

2. (a) Prove that the regular languages are closed under concatenation. [6 marks]

   (b) Explain the following properties of languages: *acceptable*, *decidable*, *recursively enumerable*, and *recursive*. Give an example in each case. [8 marks]

   (c) For each of the following languages, prove that it is regular or prove that it is not regular. [11 marks]

      i. $\{w : w \in \{a, b\}^*, w$ is the empty word, or begins with a, or contains the substring $aab\}$

      ii. $\{uxv : u, v \in \{a, b\}^*, u$ is longer than $v\}$

      iii. $\{w : w \in \{a, b\}^*, w$ has an a following every b, w has more as than bs$\}$

3. (a) Construct a (deterministic or nondeterministic) Turing machine (TM) to accept the language of correctly formatted transition functions for a nondeterministic finite automaton where $Q = \{A, B\}$, $\Sigma = \{0, 1\}$, $q_0 = A$, and $F = \{A\}$. A transition function δ will be written on the input tape as a single word with $\#$ symbols separating each entry of δ. Assume that δ will not contain multiple identical entries. Assume that the tape head is positioned at the beginning of the input initially. It is permissible for your TM to crash if it is presented with a badly formatted input. Your TM should write 'T' and halt if the input is a valid δ. Indicate which is the initial state of your TM. As examples, "$A0A\#A0B\#B0A$" (without quotes) and "" (the empty word) are valid possibilities for δ, and "$A01\#B0A$" and "$A0\#$" are not. [10 marks]

   (b) Prove or disprove each of the following. [12 marks]

      i. If a language is finite then a finite automaton can accept it.

      ii. If a language is countable then a finite automaton can accept it.

      iii. If a language is countable then a Turing machine can accept it.

   (c) What are <u>reductions</u> used for in computer theory? [3 marks]

4. (a) What does it mean for a language to be recursively enumerable but not recursive? Define two languages that are recursively enumerable but not recursive. [7 marks]

   (b) The VAREXCEED problem is defined as follows. Given a computer program P that takes three integer arguments x, y, and z, the solution to the problem is "yes" if $x > y + z$ at any instant during the execution of P. Otherwise the solution to the problem is "no." [14 marks]

      i. Use a reduction to prove the undecidability of VAREXCEED.

      ii. Consider a modification to VAREXCEED. Here, P only has one argument x, and the solution to the problem is "yes" if $x > 0$ at any instant during the execution of P. Is this modified problem decidable or undecidable? Explain.

   (c) Define precisely the <u>language</u> equivalent to the VAREXCEED problem. [4 marks]