NUI MAYNOOTH

Ollscoil na hÉireann Má Nuad

OLLSCOIL NA hÉIREANN MÁ NUAD

NATIONAL UNIVERSITY OF IRELAND MAYNOOTH

Third Computer Science & Arts Examination
Third Computer Science and Software Engineering Examination
B.Sc. (Honours) Examination
B.Sc. Computer Science and Software Engineering Examination
Master of Computer Science (Year 1) Examination
Master of Computer Science (Year 2) Examination

SEMESTER 1
2004-2005

THEORY OF COMPUTATION

PAPER CS355/SE307/CS403

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Time allowed: 2 hours

Answer ***three*** questions

All questions carry equal marks

1. Let ADD $= \{x{=}y{+}z : x, y, z \in \{1\}^*, |x| = |y| + |z|\}$ be a language over the alphabet $\Sigma = \{1, +, =\}$.

   (a) Prove that ADD is not regular. [5 marks]

   (b) Give a context-free grammar that generates ADD. [5 marks]

   (c) Construct a pushdown automaton that accepts ADD. [5 marks]

   (d) Construct a Turing machine (including full table of behaviour) that decides ADD. [6 marks]

   (e) Prove that the set of regular languages is a proper subset of the set of context-free languages. You can re-use in your proof anything you have proved already in this question. [4 marks]

2. (a) Give a regular expression that generates the language $L_{2a} = \{w : w \in \{0, 1\}^*, w$ does not begin with 11 or 00$\}$. [3 marks]

   (b) Let $L_{2b1} = \{a, ba\}$ and let $L_{2b2} = \{e, 1, 10\}$. Assume that in lexicographical order the union of the two underlying alphabets is $\{0, 1, a, b\}$ (i.e. word $a1$ comes before word aa in lexicographical order). Write out the first nine words in the lexicographical ordering of $(L^*_{2b1}) \circ L_{2b2}$. [3 marks]

   (c) Use the sequence of steps given in lectures to convert the nondeterministic finite automaton in Figure 1 on page 2 into an equivalent finite automaton. Do not remove states that will never be entered. Do not simply figure out the language and write a finite automaton from scratch. [4 marks]

   (d) Prove that the regular languages are closed under concatenation. [7 marks]

   (e) A ***useless state*** in a finite automaton is a state that is never entered on any input word. Consider the problem of testing whether a finite automaton has any useless states. Formulate this problem as a language and prove that it is decidable. [8 marks]

3. (a) Construct a finite automaton that accepts the language $L_{3a} = \{w : w \in \{a, b\}^*, w$ contains the substring baa but does not contain it at the very beginning of the word$\}$. [3 marks]

   (b) Give a regular expression that generates the language accepted by the finite automaton given in Figure 2 on page 2. [3 marks]

   (c) Prove that the complement of a nonregular language is nonregular. [4 marks]

   (d) It is claimed that finite languages are decidable and that infinite languages are undecidable. Prove or disprove each part of this claim. [5 marks]

   (e) Let $L_{3e} = \{u\#v : u, v \in \{0, 1\}^*, u^R$ is a substring of $v\}$.

      i. Prove that L_{3e} is a context-free language. [5 marks]

      ii. Can L_{3e} be accepted by a deterministic pushdown automaton? Explain. [2 marks]

      iii. If L_{3e} was modified to $L'_{3e} = \{uv : u, v \in \{0, 1\}^*, u^R$ is a substring of $v\}$ could it be accepted by a deterministic pushdown automaton? Prove your answer. Note, this language is simpler than it first appears. [3 marks]

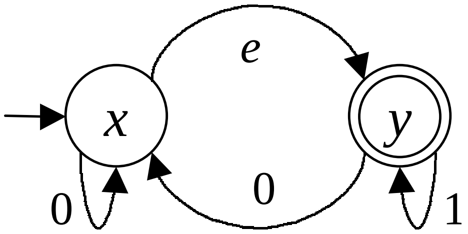

Figure 1: Nondeterministic finite automaton for question 2c.

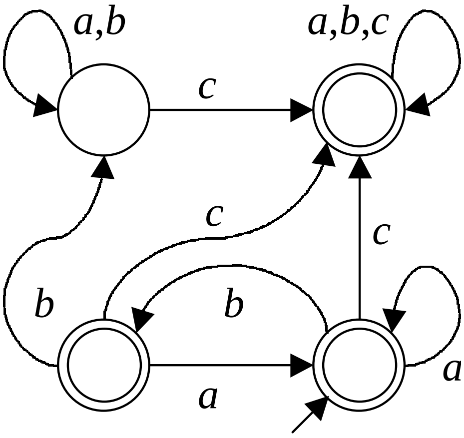

Figure 2: Finite automaton for question 3b.

4. (a) Construct a finite automaton that accepts the language $L_{4a} = \{w : w \in \{0,1\}^*, w$ contains an even number of 0s and w ends with $11\}$. [3 marks]

   (b) It is a fact that if M is a finite automaton that accepts language B, swapping the accept and non-accept states in M yields a new finite automaton that accepts $\overline{B}$.

      i. Use this fact to prove that the class of regular languages is closed under complement. [5 marks]

      ii. Prove that the above fact does not hold for nondeterministic finite automata. [5 marks]

      iii. Is the class of languages accepted by nondeterministic finite automata closed under complement? Explain. [4 marks]

   (c) Prove that $L_{4c} = \{ww : w \in \{0,1\}^*\}$ is not context-free. [8 marks]