NUI MAYNOOTH

Ollscoil na hÉireann Má Nuad

OLLSCOIL NA hÉIREANN MÁ NUAD

NATIONAL UNIVERSITY OF IRELAND MAYNOOTH

Third Computer Science & Arts Examination
Third Computer Science and Software Engineering Examination
B.Sc. (Honours) Examination
B.Sc. Computer Science and Software Engineering Examination
Master of Computer Science (Year 1) Examination
Master of Computer Science (Year 2) Examination

AUTUMN
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THEORY OF COMPUTATION

PAPER CS355/SE307/CS403

Dr. Philip Morrow, Prof. Ronan Reilly, Mr. Tom Naughton

Time allowed: 2 hours

Answer ***three*** questions

All questions carry equal marks

1. (a) Let ADD $= \{x{=}y{+}z : x, y, z \in \{1\}^*, |x| = |y| + |z|\}$ be a language over the alphabet $\Sigma = \{1, +, =\}$. Construct a Turing machine (including full table of behaviour) that decides ADD. [8 marks]

   (b) Prove that the regular languages are closed under concatenation. [7 marks]

   (c) A ***useless state*** in a finite automaton is a state that is never entered on any input word. Consider the problem of testing whether a finite automaton has any useless states. Formulate this problem as a language and prove that it is decidable. [10 marks]

2. (a) Let $L = \{w : w \in \{a, b\}^*, w$ does not contain aa as a substring$\}$. Construct two finite automata, one deterministic and the other nondeterministic, that recognise L. [6 marks]

   (b) Give definitions for two context-free languages, one of which <u>can</u> be recognised by a deterministic pushdown automaton and one that <u>can not</u> be recognised by a deterministic pushdown automaton. [4 marks]

   (c) It is claimed that finite languages are decidable and that infinite languages are undecidable. Prove or disprove each part of this claim. [5 marks]

   (d) The complement of a regular language is regular. The complement of a non-regular language is nonregular. Therefore, it is claimed that the language $L = \{uv : u, v \in \{a, b\}^*, u$ is not equal to $v\}$ is nonregular. Argue in support of, or against, this claim. [10 marks]

3. (a) Expand the languages defined by the following expressions. Note, e denotes the empty word, $\circ$ denotes concatenation, $\emptyset$ denotes the empty set, and 2^L denotes the power set of L. [7 marks]

   i. $\emptyset \cup \{aa, ab\}$
   ii. $\{e\}^*$
   iii. $\emptyset^*$
   iv. $\emptyset \circ \{a, b, c\}$
   v. 2^L, where the language $L = \{e, ab\}$
   vi. the regular expression $(0 \cup e)1$
   vii. the context-free grammar $S \rightarrow SSS|SS|e$

   (b) Is it possible to enumerate the set of all words over a finite alphabet? Prove your answer. [5 marks]

   (c) For each of the following languages, prove that it is regular or prove that it is not regular. [13 marks]

   i. $\{w : w \in \{a, b\}^*, w$ is the empty word, or begins with a, or contains the substring $aab\}$
   ii. $\{wxw^R : w \in \{a, b\}^*\}$
   iii. $\{uv : u, v \in \{a, b\}^*, u$ is longer than $v\}$

4. (a) The *type* of the value of an arithmetic expression (such as $3 \times 4 + 5$) is a number (17 in this case). What is the *type* of the value of a regular expression? What is the *type* of the value of a context-free grammar? [4 marks]

(b) For each of the following context-free languages design both a pushdown automaton to recognise it and a context-free grammar to generate it.

i. $\{vw : v \in \{a, b\}, w \in \{a, b\}^*, w$ contains twice as many as as bs if $v = a, w$ contains twice as many bs as as if $v = b\ \}$ [10 marks]

ii. $\{w : w \in \{a, b\}^*, w$ contains no less than two as$\}$ [4 marks]

iii. $\{w : w \in \{a, b\}^*, w$ contains more as than bs$\}$ [7 marks]