NUI MAYNOOTH

Ollscoil na hÉireann Má Nuad

# OLLSCOIL NA hÉIREANN MÁ NUAD

# THE NATIONAL UNIVERSITY OF IRELAND MAYNOOTH

**First Arts Examination**
**First Computer Science And Software Engineering Examination**

**Year 1**

**SEMESTER 1**
**2004-2005**

# DISCRETE STRUCTURES 1
# PAPER SE119

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Time allowed: 2 hours

Answer ***three*** questions

**All questions** carry equal marks

**1** (a) Given the following propositions P, Q and R: **[4 marks]**

P: $\forall$ x, x+1 > x
Q: $\forall$ x, $x^2 > 0$
R: $\forall$ x, $(x+2)^2 = x^2+4$

Determine the truth values for each of the following compound propositions:

- NOT ((P OR Q) AND NOT R)
- (P AND NOT Q) IF AND ONLY IF R

(b) Verify each of the following laws of De Morgan: **[6 marks]**

- NOT(P OR Q) ≡ NOT P AND NOT Q
- NOT(P AND Q) ≡ NOT P OR NOT Q

(c) Prove the following propositions. Clearly state the proof strategy used in your solution: **[15 marks]**

- If $n^2$ is odd, then n is odd.
- If c|a and c|b, then c|(am+bn) for any integers m and n.
- For any integer x and any prime number p, if p divides x, then p does not divide x+1.

**2** (a) In a survey of 96 students, the following results were recorded: **[6 marks]**

There are 43 students that live on campus.
There are 38 women in the group.
There are 18 seniors in the group.
There are 11 seniors that live on campus.
There are 10 senior women.
There are 7 women that live on campus.
There are 6 senior women that live on campus.

Provide solutions to the following questions explaining your solution in each case:

- How many women do not live on campus?
- How many senior men are there?
- How many non-senior men do not live on campus?

(b) For each of the following **relations**, determine whether the relation is a function. If the relation is a **function**, determine whether the function is **injective** and/or **surjective**. For those functions that are bijections determine the **inverse** function. **[6 marks]**

- `A = {v,w,x,y,z}, B= {1,2,3,4,5}`
  `R = {(v,2),(w,1),(x,3),(y,5)}`

- `A = {1,2,3,4,5}, B= {1,2,3,4,5}`
  `R = {(1,2), (2,3), (3,4), (4,5), (5,1)}`

(c) Describe **Euclids Algorithm**. Illustrate how you would use Euclid's Algorithm to calculate the **greatest common divisor** of `117` and `65`. **[6 marks]**

(d) Consider the function `f(x,n) = x mod n`. What is the **domain** of `f`? What is the **range** of `f`? Let `g(n) = f(209,n)`. Evaluate `g(g(g(g(7))))`. **[7 marks]**

**3** (a) Write down each step in the **evaluation** of `f(13)` where `f` has the following recursive definition: **[5 marks]**

```
f(1) = 1
f(n) = f(ceiling(n/2)) + n.
```

(b) Write a **recursive definition** for the following functions: **[5 marks]**

- `f: N → N defined by f(n) = 1 + 5 + 9 + … + (4n+1).`
- `f: String → String` that replaces each occurrence of the letter `a` by `b` in a string over the alphabet `{a,b,c}.`

(c) Write an **inductive proof** to show the following statement is true for all the natural numbers `n >= 1`: **[6 marks]**

$1^2 + 2^2 + 3^2 + \ldots + n^2 = (n(n+1)(2n+1))/6$

(d) Write down a **recursive definition** that prints the elements of a list of Integers in reverse order. You may assume the existence of a print procedure that prints a single Integer. Provide an **inductive proof** to show that the function definition is correct for all input lists. **[9 marks]**

**4** (a) Prove each of the following statements using **equivalence laws**. Clearly state which law is used at each stage of the proof: **[8 marks]**

- $(A \rightarrow B) \equiv \neg (A \wedge \neg B)$
- $(A \wedge B \rightarrow C) \rightarrow (A \wedge C \rightarrow B) \equiv A \wedge C \rightarrow B$

(b) Use **Quines method** to show that the following: **[6 marks]**

- $(A \rightarrow B) \wedge (B \rightarrow \neg A) \rightarrow A$ is a contingency
- $(A \vee B) \wedge (A \rightarrow C) \wedge (B \rightarrow D) \rightarrow (C \vee D)$ is a tautology

(c) Assume that x and y are real numbers and let P(x,y) denote the predicate x + y = 0. Express each of the following propositions in words and determine their truth value: **[5 marks]**

- $\forall x(\exists y(P(x,y)))$
- $\exists x(\forall y(P(x,y))$

(d) Formalise each of the following English sentences where $P(x)$ means $x$ is a person, $S(x)$ means $x$ can't swim, and $F(x)$ means $x$ is a fish. **[6 marks]**

- All fish can swim. John can't swim. Therefore, John is not a fish.
- Some people can't swim. All fish can swim. Therefore, there is some person who is not a fish.