NUI MAYNOOTH

Ollscoil na hÉireann Má Nuad

OLLSCOIL NA hÉIREANN MÁ NUAD

THE NATIONAL UNIVERSITY OF IRELAND MAYNOOTH

First Arts Examination
First Computer Science And Software Engineering Examination

Year 1

AUTUMN EXAMINATION
2004-2005

DISCRETE STRUCTURES 1
PAPER SE119

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Time allowed: 2 hours

Answer ***three*** questions

All questions carry equal marks

1 (a) Assuming that proposition P is true, proposition Q is false, and proposition R is true, find the truth value of each of the following compound propositions: **[6 marks]**

- IF (P AND Q) THEN R
- IF (P OR Q) THEN NOT R
- P AND (IF Q THEN R)

(b) Verify each of the following equivalence laws: **[4 marks]**

- $A \rightarrow B \equiv \neg A \vee B$
- $\neg (A \rightarrow B) \equiv A \wedge (B \rightarrow \text{False})$

(c) Prove the following propositions. Clearly state the proof strategy used in your solution: **[15 marks]**

- For all integers n, n is odd if and only if n-1 is even
- For all real numbers x and y, if $x+y \geq 2$, then either $x \geq 1$ or $y \geq 1$.
- There exists a prime p such that $2^p - 1$ is not a prime.

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2 (a) A television poll of 151 people found that: **[4 marks]**

- 68 watched “Law and Disorder”
- 61 watched “The East Wing”
- 52 watched “ The Tenors”
- 16 watched both “Law and Disorder” and “The East Wing”
- 25 watched both “Law and Disorder” and “The Tenors”
- 30 watched “Law and Disorder” and no other show
- 26 watched none of these shows.

How many of the people surveyed watched all three shows?
Clearly, explain your solution.

(b) Each of the following functions are **bijective** on the specified domain A. **[9 marks]**

i) Define the term bijective.
ii) Find the inverse of each function below.
iii) Explain your solution in each case.

- f(x) = 4x + 2, A = set of real numbers
- f(x) = 3 + 1/x, A = set of non zero real numbers
- f(x) = (2x + 3) mod 7, A = N_7

(c) Calculate the **fixed point** of the following function: **[6 marks]**

f(x) = (2x + 3) mod 7, defined on N_7

Explain the term fixed point in your solution.

(d) Let f and g be functions from the positive real numbers to the positive real numbers defined by the equations: **[6 marks]**

$f(x) = \lfloor 2x \rfloor$
$g(x) = x^2$

Calculate the compositions $f \circ g$ and $g \circ f$.

3 (a) Write down each step in the **evaluation** of `f(4)` where `f` has the following recursive definition: **[4 marks]**

```
f(0) = 0
f(n) = f(n-1) + n.
```

(b) Write a **recursive definition** for the following functions: **[6 marks]**

- `f: N → N defined by f(n) = 1(2) + 2(3) + 3(4) + … + n(n+1).`
- `f: String → String` that removes each occurrence of the letter `a` from any string over the alphabet `{a,b,c}.`

(c) Write an **inductive proof** to show the following statement is true for all the natural numbers `n >= 1`: **[6 marks]**

```
1(1!) + 2(2!) + ... + n(n!)  = (n+1)! –1
```

(d) Write down a **recursive definition** that prints the even elements of a list of Integers. You may assume the existence of a `print` procedure that prints a single Integer. Provide an **inductive proof** to show that the function definition is correct for all input lists. **[9 marks]**

4 (a) Use **Quines method** to show that the following: **[6 marks]**

- $(Q \rightarrow P) \wedge (P \rightarrow \neg Q) \rightarrow Q$ is a contingency
- $(Q \vee P) \wedge (Q \rightarrow R) \wedge (P \rightarrow S) \rightarrow (R \vee S)$ is a tautology

(b) Use equivalences to transform each of the following into Conjunctive Normal Form (CNF): **[6 marks]**

- $(P \rightarrow Q) \rightarrow P$
- $Q \wedge \neg P \rightarrow P$
- $P \rightarrow Q \wedge R$

(c) Let P(x) denote the statement " x is a professional athlete" and let Q(x) denote the statement " x plays soccer". The domain is the set of all people. Write each of the following propositions in English: **[9 marks]**

- $\forall x\ (P(x) \rightarrow Q(x))$
- $\exists x\ (P(x) \wedge Q(x))$
- $\forall x\ (P(x) \vee Q(x))$

Write the negation of each of these propositions, both in symbols and in words. Simplify the negated propositions as much as is possible.

(d) Formalise each of the following English sentences where P(x) denotes the statement "x is an accountant", Q(x) denotes the statement "x owns a Porsche": **[4 marks]**

- All accountants own Porsches
- Some accountants own Porsches
- All owners of Porsches are accountants
- At least one person who owns a Porsche is an accountant