Rules, Definitions, and Theorems that can be applied to any question

Not ($\overline{P}$, $\neg P$) truth table:

P	$\neg P$
T	F
F	T

And truth table:

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Or truth table:

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Implication ($P \rightarrow Q$) truth table:

P	Q	if P then Q
T	T	T
T	F	F
F	T	T
F	F	T

If and only if (iff) truth table:

P	Q	P iff Q
T	T	T
T	F	F
F	T	F
F	F	T

Equivalence laws

not (A and B) $\equiv$ (not A) or (not B)
not (A or B) $\equiv$ (not A) and (not B)
A and (B or C) $\equiv$ (A and B) or (A and C)
A or (B and C) $\equiv$ (A or B) and (A or C)
if A then $B \equiv$ if not B then not A
if A then $B \equiv$ (not A) or B

Divisibility and prime numbers

$\mathbb{Z}$ is the set of whole numbers (includes negative numbers and 0) called integers.
$\mathbb{N}$ is the set of nonnegative whole numbers (includes 0) called naturals.
$\mathbb{R}$ is the set of real numbers.
The set of prime numbers is $\{x : x \in \mathbb{N}, x \text{ has exactly two factors: } 1 \text{ and } x\}$. Therefore, 2 is the smallest prime number.
Integer d divides integer n (written $d|n$) if $d \neq 0$ and there is a $k \in \mathbb{Z}$ such that $n = dk$.

Proof techniques

- Proof by exhaustive techniques. To prove a statement is true by exhaustive checking one must prove that the statement is true for every possible value.

- Proof by counter example. To prove a statement false by a counter example one simply finds a single value for which the statement is false.

- Conditional proof (direct proof). One uses a conditional proof if one is asked to prove a statement of the form "if A then B." It requires one to first assume that A is true. Then one makes a statement consisting of A and any other known facts. If using the valid rules of logic one can derive B from this statement then this proves that the "if A then B" statement must be true.

- Conditional proof (proving the contrapositive). One might also prove a statement of the form "if A then B" by proving the contrapositive: proving that "if not B then not A." Here one would assume that B is false and then continue as in a direct proof to derive that A is false.

- Proof by contradiction. A proof by contradiction would involve one assuming the statement is false and then using this fact and any other known facts to derive a contradiction (i.e. such as deriving that an integer is both even and odd, or deriving that an element is both in and out of a particular set).

- If and only if proof. Proving a statement "A if and only if B," sometimes abbreviated to "A iff B," would require one to prove both "if A then B" and "if B then A."

Sets and tuples

A set A is a subset of B, written $A \subset B$, if for every $x \in A$ it is true that $x \in B$.
For every set A, both $\emptyset \subset A$ and $A \subset A$ are true.
A set A is equal to B, written $A = B$, if $A \subset B$ and $B \subset A$.
A set A is a proper subset of B if $A \subset B$ and $A \neq B$.
The union of two sets A and B is $A \cup B = \{x : x \in A \text{ or } x \in B\}$.
The intersection of two sets A and B is $A \cap B = \{x : x \in A \text{ and } x \in B\}$.
The difference of two sets A and B is $A - B = \{x : x \in A \text{ and } x \notin B\}$.
The power set 2^A [sometimes written power(A)] of a set A is $\{x : x \subset A\}$.
A tuple is an ordered collection of objects that can contain duplicates.
Tuples are written using parentheses $(\dots)$ rather than the braces $\{\dots\}$ used for sets.
The cross product of two sets A and B is $A \times B = \{(a, b) : a \in A, b \in B\}$.

Functions

$f : A \to B$ is a function called f mapping elements from domain A to co-domain B.
The range of $f : A \to B$ is $\{f(a) : a \in A\}$.
$f : A \to B$ is equal to $g : A \to B$ if $f(a) = g(a)$ for all $a \in A$.
abs: $\mathbb{R} \to \mathbb{R}$ is defined abs$(x) =$ if $x \geq 0$ then x else $-x$.
floor: $\mathbb{R} \to \mathbb{Z}$, written $\lfloor x \rfloor$, is defined as the largest integer $\leq x$.
ceiling: $\mathbb{R} \to \mathbb{Z}$, written $\lceil x \rceil$, is defined as the smallest integer $\geq x$.
mod: $\mathbb{Z} \times (\mathbb{N} - \{0\}) \to \mathbb{N}$ is defined mod$(a, b) = a - b\lfloor a/b \rfloor$.