1. (a) What does it mean if a Hoare triple evaluates to true, and how can this be used to prove that a computer program is correct? Make reference to each of the three parts of the triple in your answer. [5 marks]

(b) Prove the correctness or incorrectness of each of the following computer programs. You must use the technique based on calculating the most general (or weakest) precondition.
   i. \( \{ y < 0 \} x := 5; x := x + y \{ x < 10 \} \)
   ii. \( \{ x > 0 \} x := x \times x \{ x^4 = 10 \} \)
   iii. \( \{ x > 5 \} x := 5; x := x + 1 \{ x > 5 \} \)

2. (a) Let a function \( f : \mathbb{N} \to \mathbb{N} \) be defined as \( f(x) = \text{floor}(\sqrt{x}) \). [15 marks]
   i. Write an expression for the equivalence classes in the partition of \( \mathbb{N} \) induced by the kernel relation of \( f \).
   ii. Write out the first four equivalence classes.

(b) Prove the correctness of the following computer program. [10 marks]
   \( \{ x > 0 \} \text{if } x > 5 \text{ then } x := 5 \{ 0 \leq x \leq 5 \} \)
3. (a) Why are language acceptance (language recognition) problems of interest to computer scientists? [5 marks]

(b) Let the set $X$ be defined as $X = \{ x | x \in \mathbb{R} \land 100x \in \mathbb{N} \}$. For example, $5.21 \in X$ and $0.99 \in X$. Prove that $X$ is countable. [5 marks]

(c) Prove the correctness of the following computer program. [15 marks]

\[
\begin{align*}
\{ x > 0 \} &
y := 17; \\
\{ x > 0 \land y \leq 17 \} &// P \\
\textbf{while} & \ x > y \ \textbf{do} \\
\textbf{\ }& x := x - y \\
\textbf{od} \\
\{ 0 < x \leq 17 \}
\end{align*}
\]

4. (a) Given the binary relation $R = \{ (1, 2), (3, 1), (3, 2), (2, 4) \}$ over $\{1, 2, 3, 4\}$, construct each of the following relations. [9 marks]

i. $r(R)$ (reflexive closure)

ii. $s(R)$ (symmetric closure)

iii. $t(R)$ (transitive closure)

(b) Given some finite alphabet $\Sigma$, the set of all words over $\Sigma$ is countable and is denoted $L = \Sigma^*$. Prove that the set of all languages over $\Sigma$, denoted $2^L$ or $\text{power}(L)$, is uncountable. [16 marks]
SE120 Axioms and Theorems

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Implication truth table:

Assignment axiom (AA): \[ Q(x/t) \equiv t \{ Q \} \]

Consequence Rule: \[ \frac{P \rightarrow R \quad \{ R \} S \{ Q \}}{P S \{ Q \}} \]

Composition Rule: \[ \frac{\{ P \} S_1 \{ R \} \quad \{ R \} S_2 \{ Q \}}{\{ P \} S_1 ; S_2 \{ Q \}} \]

If-Then Rule: \[ \frac{\{ P \land C \} S \{ Q \} \quad P \land \neg C \rightarrow Q}{\{ P \} \text{ if } C \text{ then } S \{ Q \}} \]

If-Then-Else Rule: \[ \frac{\{ P \land C \} S_1 \{ Q \} \quad \{ P \land \neg C \} S_2 \{ Q \}}{\{ P \} \text{ if } C \text{ then } S_1 \text{ else } S_2 \{ Q \}} \]

While Rule: \[ \frac{\{ P \land C \} S \{ P \}}{\{ P \} \text{ while } C \text{ do } S \{ P \land \neg C \}} \]

Selected theorems that can be quoted without proof:
1. The union of any finite number of countable sets is a countable set
2. The cross product of any finite number of countable sets is a countable set
3. The intersection of any finite number of countable sets is a countable set
4. The power set of a finite set is a finite set