1. (a) Expand the languages defined by the following expressions. Note, $e$ denotes the empty word, $\cdot$ denotes concatenation, $\emptyset$ denotes the empty set, and $2^{L}$ denotes the power set of $L$.
   i. $\emptyset \cup \{aa, ab\}$
   ii. $\{e\}^*$
   iii. $\emptyset^*$
   iv. $\emptyset \cdot \{a, b, c\}$
   v. $2^L$, where the language $L = \{e, ab\}$
   vi. the regular expression $(\emptyset \cup e)1$
   (b) Prove that the regular languages are closed under concatenation.
   (c) Can you enumerate the set of all words over a finite alphabet? Prove your answer.
   (d) Explain the following properties of languages: acceptable, decidable, recursively enumerable, and recursive. Give an example in each case.

2. Define a language that is recursively-enumerable and nonrecursive. Prove that your language has both properties. Full marks will be awarded for an unambiguous definition and a detailed proof.
3. (a) For each of the following languages, prove that it is regular or prove that it is not regular. [13 marks]
   i. \( \{ w : w \in \{a, b\}^*, w \text{ is the empty word or contains the substring } aab \} \)
   ii. \( \{ w w : w \in \{a, b\}^* \} \)
   iii. \( \{ uv : u, v \in \{a, b\}^*, u \text{ is not equal to } v \} \)

(b) Prove that the set of regular languages is a proper subset of the set of the context-free languages. The only theorems you may use (if you wish to) are those you have proved from part (a) of this question and the following:
   - a language is regular iff it is accepted by a finite automaton
   - a language is context-free iff it is accepted by a pushdown automaton. [12 marks]

4. (a) Define any decision problem relating to finite automata. State whether this problem would be decidable or not by a Turing machine. Prove your answer for full marks. [8 marks]

(b) What does a reduction \( A \leq B \) between two problems \( A \) and \( B \) establish about the relative computability of \( A \) and \( B \)? What does a polynomial reduction establish about the relative computational complexity of \( A \) and \( B \)? [2 marks]

(c) Use a reduction to prove the undecidability of the \textsc{Varinequality} problem. \textsc{Varinequality} is defined as follows. Given a computer program \( P \) that takes no input, and two integer variables \( A \) and \( B \) declared in \( P \), will the value in \( B \) ever exceed the value in \( A \) during the execution of \( P \)? [7 marks]

(d) Use a reduction to prove the \( \mathcal{NP} \)-completeness of the \textsc{Hittingset} problem. You may assume that \textsc{Hittingset} is in \( \mathcal{NP} \), and that 3-SAT is \( \mathcal{NP} \)-complete. \textsc{Hittingset} is defined as follows. Given a system \( \{A_1, \ldots, A_m\} \) of finite sets and a natural number \( k \), does any set with no more than \( k \) elements exist that intersects every \( A_i \)? [8 marks]