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Ollscoil na hÉireann Má Nuad

IRISH RESEARCH COUNCIL
An Chomhairle um Thaighde in Éirinn

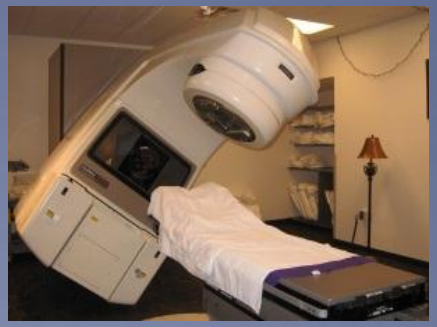
A LOGICAL FRAMEWORK FOR INTEGRATING SOFTWARE MODELS VIA REFINEMENT

Marie Farrell

Supervisors: Dr. Rosemary Monahan & Dr. James Power

MOTIVATION

Ariane 5
€350,000,000



Therac-25
3 Fatalities

$312 BILLION

BACKGROUND

- Formal software engineering is a set of mathematically grounded techniques for the specification, development and verification of software and hardware systems.
- A formal specification is the exact definition in mathematical notation of what the system is required to do (and not do).

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EVENT B

- The Event B formal specification language is used in the verification of safety critical systems

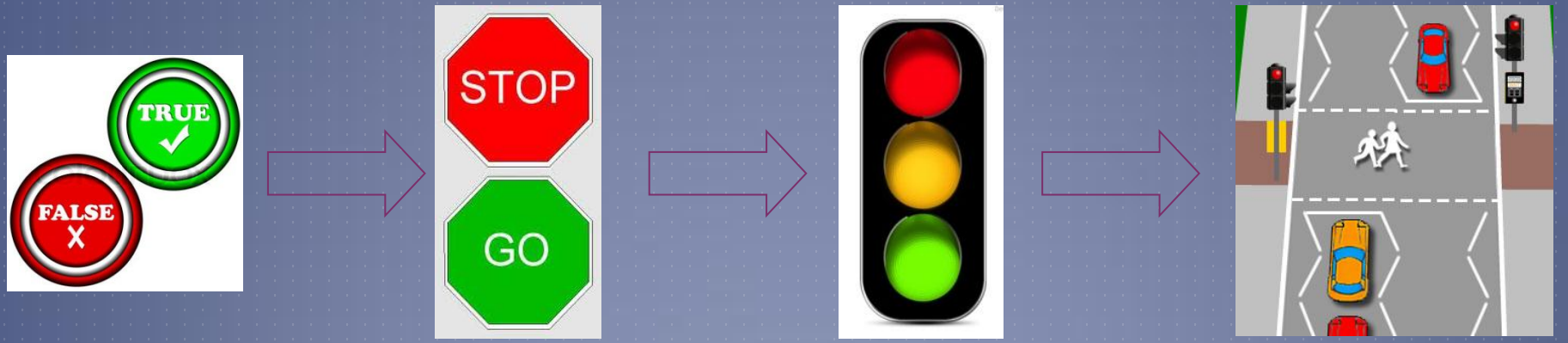


- Event B models are an instance of the specification

Machine	Context
variables	carrier sets
invariants	constants
events	axioms

REFINEMENT

- Refinement provides a way for us to model software at different levels of abstraction



SOCIAL NETWORK

```
MACHINE
    mac1
SEES
    ctx1
VARIABLES
    person
    rawcontent
    content
    owner
INVARIANTS
    inv1 : person ⊆ PERSON
    inv2 : rawcontent ⊆ RAWCONTENT
    inv3 : content ∈ rawcontent ↔ person
    inv4 : owner ∈ rawcontent → person
EVENTS
    INITIALISATION ≙
        STATUS
        ordinary
BEGIN
    act1 : person ≔ ∅
    act2 : rawcontent ≔ ∅
    act3 : content ≔ ∅
    act4 : owner ≔ ∅
END

    transmit ≙
        STATUS
        ordinary
ANY
    rc
    pe
WHERE
    grd1 : rc ∈ rawcontent
    grd2 : pe ∈ person
    grd3 : rc ↦ pe ∉ content
THEN
    act1 : content ≔ content ∪ {rc ↦ pe}
END

END
```

```
MACHINE
    mac2
REFINES
    mac1
SEES
    ctx1
    ctx2
VARIABLES
    person
    rawcontent
    content
    owner
    visible
    viewpermission
INVARIANTS
    inv1 : visible ∈ rawcontent ↔ person
    inv2 : viewpermission ∈ person ↔ person
EVENTS
    INITIALISATION ≙
        extended
            STATUS
        ordinary
BEGIN
    act1 : person ≔ ∅
    act2 : rawcontent ≔ ∅
    act3 : content ≔ ∅
    act4 : owner ≔ ∅
    act5 : visible ≔ ∅
    act6 : viewpermission ≔ ∅
END

    transmit ≙
        STATUS
        ordinary
REFINES
    transmit
ANY
    rc
    pe
WHERE
    grd1 : rc ∈ rawcontent
    grd2 : pe ∈ person
    grd3 : rc ↦ pe ∉ content
THEN
    act1 : visible ≔ visible ∪ {rc ↦ pe}
    act2 : viewpermission ≔ viewpermission ∪ {owner(rc) ↦ pe}
END
```

PROBLEM

- Different formalisms do not integrate well e.g. Event B models the specification it does nothing for the implementation and its proofs are not easily transferable to other formalisms

SOLUTION

- Establish a theoretical framework within which refinement steps, and their associated proof obligations, can be shared between different formalisms
- Hypothesis: the theory of institutions can provide this framework and, we will construct an institution based specification of the Event B formalism

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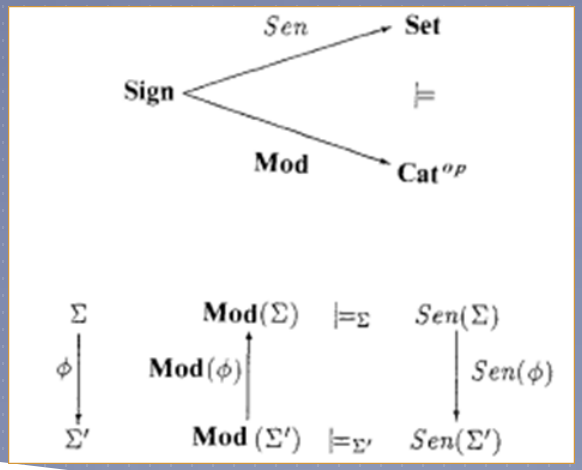
CATEGORY THEORY / INSTITUTIONS

- Category Theory is a special branch of Mathematics that allows us not only to describe objects but also to investigate the relationships between them
- Institutions are an application of category theory that allow us to relate the syntactic and semantic structures of different formal languages



ALFRED TARSKI

1901 - 1983

- Polish mathematician/logician
- Born Alfred Tajtelbaum
- Travelled to the USA in 1939
- Harvard, City College of New York, Princeton and University of California at Berkely

"Snow is white" is true if and only if snow is white

The concept of truth in formalized languages

P is true if and only if P

UBER EINIGE FUNDAMENTALE BEGRIFFE DER METAMATHEMATIK

MARIE FARRELL

ON SOME FUNDAMENTAL CONCEPTS OF METAMATHEMATICS

- Formalized deductive disciplines form the field of research of metamathematics
- These disciplines are regarded as sets of sentences
- The set of all sentences is denoted by 'S'
- From the sentences of an set X certain other sentences can be obtained using rules of inference
- These sentences are called the *'consequences'* of X
- The set of all consequences is denoted by 'Cn(X)'

ON SOME FUNDAMENTAL CONCEPTS OF METAMATHEMATICS

1930

- Axiom 2:

$$If\ X \subseteq S, then\ X \subseteq Cn(X) \subseteq S$$

- Axiom 3:

$$If\ X \subseteq S, then\ Cn\big(Cn(X)\big) = Cn(X)$$

- Axiom 4:

$$If\ X \subseteq S, then\ Cn(X) = \sum_{Y \subseteq X\ and\ |Y| < \aleph_0} Cn(Y)$$

Π - INSTITUTIONS

- Alternative to institution – replacing the notions of model and satisfaction by Tarski's consequence operator

- Definition:
  - A π-institution is a triple (Sign, φ, $\{Cn_\Sigma\}_{\Sigma:Sign}$) consisting of
    1. A category Sign (of signatures)
    2. A functor φ:Sign -> Set (set of formulae over each signaure)
    3. For each object Σ of Sign, a consequence operator Cn_Σ defined in the power set of φ(Σ) satisfying for each A, B ⊆ φ(Σ) and μ: Σ -> Σ

(RQ1) $A \subseteq Cn_\Sigma(A)$ (Extensiveness)

(RQ2) $Cn_\Sigma(Cn_\Sigma(A)) = Cn_\Sigma(A)$ (Idempotence)

(RQ3) $Cn_\Sigma(A) = \bigcup_{B \subseteq A, B\ finite} Cn_\Sigma(B)$ (Compactness)

(RQ4) $\varphi(\mu)(Cn_\Sigma(A)) \subseteq Cn_{\Sigma'}(\varphi(\mu)(A)$ (Structurality)

Tarski: Axioms 2, 3 &4

CONCLUSION

- Tarski provided the foundations for π-institutions
- Work to date:
  - Denotational Semantics
  - Communicating Sequential Processes (Hoare)
  - Tarski
  - Consequence
  - Category Theory/Institutions/π-institutions

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