A LOGICAL FRAMEWORK FOR INTEGRATING SOFTWARE MODELS VIA REFINEMENT

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MOTIVATION

Ariane 5
€350,000,000

Therac-25
3 Fatalities

$312 BILLION

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BACKGROUND

- Formal software engineering is a set of mathematically grounded techniques for the specification, development and verification of software and hardware systems.
- A formal specification is the exact definition in mathematical notation of what the system is required to do (and not do).

PROBLEM

- Different formalisms do not integrate well

SOLUTION

- Establish a theoretical framework within which refinement steps, and their associated proof obligations, can be shared between different formalisms
- Hypothesis: the theory of institutions can provide this framework and, we will construct an institution based specification of the Event B formalism

RESEARCH QUESTIONS

1. Can the theory of institutions ensure the accuracy of the translation between Event-B and other specification formalisms?

2. Can this theory allow us to investigate proof obligations generated by Event-B in different formalisms?

EVENT B

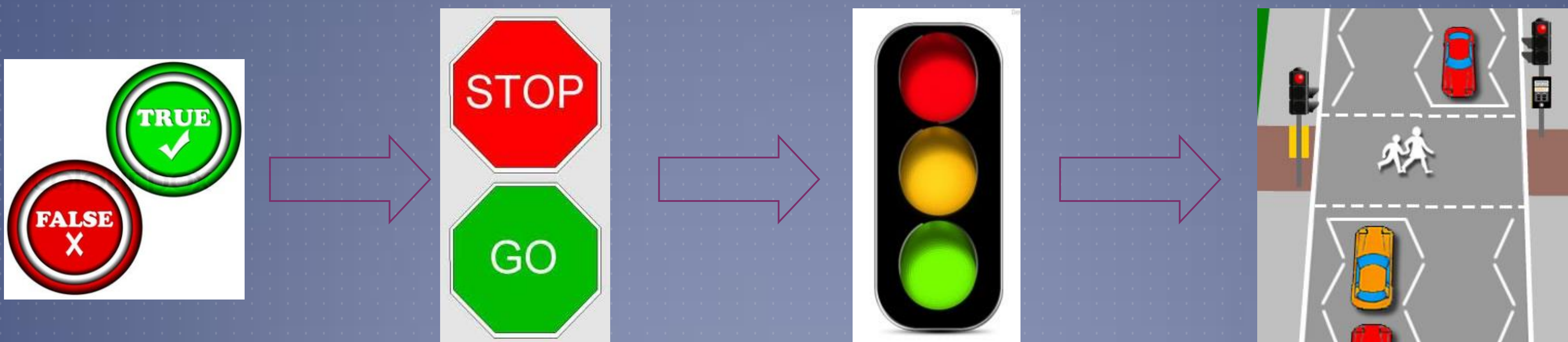
- The Event B formal specification language is used in the verification of safety critical systems

- Event B models are an instance of the specification

Machine	Context
variables	carrier sets
invariants	constants
events	axioms

REFINEMENT

- Refinement provides a way for us to model software at different levels of abstraction



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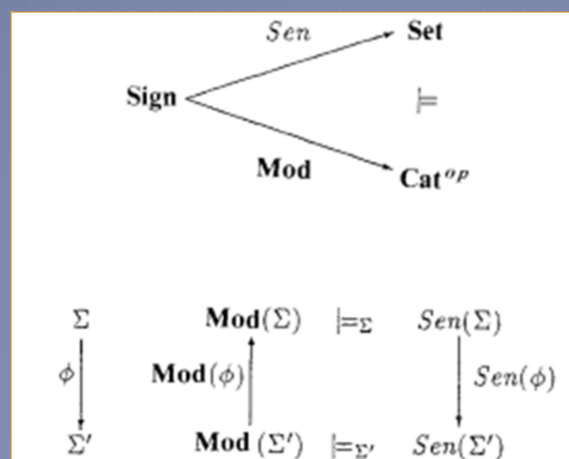
SOCIAL NETWORK

```
MACHINE
    mac1
SEES
    ctx1
VARIABLES
    person
    rawcontent
    content
    owner
INVARIANTS
    inv1 : person ⊆ PERSON
    inv2 : rawcontent ⊆ RAWCONTENT
    inv3 : content ∈ rawcontent ↔ person
    inv4 : owner ∈ rawcontent → person
EVENTS
    INITIALISATION ≙
        STATUS
      ordinary
BEGIN
      act1 : person := ∅
      act2 : rawcontent := ∅
      act3 : content := ∅
      act4 : owner := ∅
END

    transmit ≙
        STATUS
      ordinary
ANY
      rc
      pe
WHERE
      grd1 : rc ∈ rawcontent
      grd2 : pe ∈ person
      grd3 : rc ↦ pe ∉ content
THEN
      act1 : content := content ∪ {rc ↦ pe}
END

END
```

```
MACHINE
    mac2
REFINES
    mac1
SEES
    ctx1
    ctx2
VARIABLES
    person
    rawcontent
    content
    owner
    visible
    viewpermission
INVARIANTS
    inv1 : visible ∈ rawcontent ↔ person
    inv2 : viewpermission ∈ person ↔ person
EVENTS
    INITIALISATION ≙
      extended
        STATUS
      ordinary
BEGIN
      act1 : person := ∅
      act2 : rawcontent := ∅
      act3 : content := ∅
      act4 : owner := ∅
      act5 : visible := ∅
      act6 : viewpermission := ∅
END

    transmit ≙
        STATUS
      ordinary
REFINES
      transmit
ANY
      rc
      pe
WHERE
      grd1 : rc ∈ rawcontent
      grd2 : pe ∈ person
      grd3 : rc ↦ pe ∉ content
THEN
      act1 : visible := visible ∪ {rc ↦ pe}
      act2 : viewpermission := viewpermission ∪ {owner(rc) ↦ pe}
END
```

CATEGORY THEORY / INSTITUTIONS

- Category Theory is a special branch of Mathematics that allows us not only to describe objects but also to investigate the relationships between them
- Institutions are an application of category theory that allow us to relate the syntactic and semantic structures of different formal languages



Π - INSTITUTIONS

- Alternative to institution – replacing the notions of model and satisfaction by Tarski's consequence operator

- Definition:
  - A π-institution is a triple (Sign, φ, $\{Cn_\Sigma\}_{\Sigma:Sign}$) consisting of
    1. A category Sign (of signatures)
    2. A functor φ:Sign -> Set (set of formulae over each signature)
    3. For each object Σ of Sign, a consequence operator Cn_Σ defined in the power set of φ(Σ) satisfying for each A, B ⊆ φ(Σ) and μ: Σ -> Σ

(RQ1) $A \subseteq Cn_\Sigma(A)$	(Extensiveness)
(RQ2) $Cn_\Sigma(\ Cn_\Sigma(A)\ ) = Cn_\Sigma(A)$	(Idempotence)
(RQ3) $Cn_\Sigma(A) = \bigcup_{B \subseteq A, B\ finite} Cn_\Sigma(B)$	(Compactness)
(RQ4) $\varphi(\mu)(Cn_\Sigma(A)) \subseteq Cn_{\Sigma'}(\varphi(\mu)(A)$	(Structurality)

REFINEMENT CALCULUS

- Refinement calculus is a notation and a set of rules for deriving programs from their specifications
- Refinement calculii are an extension of Dijkstra's language of guarded commands and both specification and implementation occur within the same formalism
- There are three main theories of refinement:
    1. Carroll Morgan
    2. Ralph-Johan Back
    3. Joseph Morris

MORGAN VS BACK VS MORRIS

- The definition of what constitutes refinement appears to be the same in all calculi
- The rules, however, are slightly different: Morgan is the only one to use miracles
- Back's refinement calculus is much more theoretical that that of Morgan using lattice and category theory as its underlying mathematical basis
- Morris extended Back's refinement calculus to include the notion of prescription
- Since the meaning of what is a valid refinement stays the same then regardless of how it is carried out we should always be able to refine a given specification to an implementation that is semantically consistent across all calculi.

GENERAL THEORY OF REFINEMENT
- REEVES AND STREADER 2008

- The general model takes as primitive:
  1. A set of entities: the specifications and implementations we wish to develop by refinement
  2. A set of contexts: the environment with which the entities interact
  3. A user formalised by defining the set of observations that can be made when an entity is executed in a given context
- The general definition of refinement is parameterised by a set Ξ of possible contexts and a function O which determines what can be observed
- The concrete entity **C** is a refinement of an abstract entity **A** when no user of **A** could observe if they were given **C** in place of **A**.

DEFINITION

- Let Ξ be a set of contexts each of which entities **C** and **A** can communicate privately with, and O be a function which returns a set of traces, each trace being what a user observes of an execution then:

$$\boldsymbol{A} \sqsubseteq_{\Xi,\mathbf{O}} \boldsymbol{C} \triangleq \forall x \in \Xi.\, O([\boldsymbol{C}]_x) \subseteq O([\boldsymbol{A}]_x)$$

- Since general refinement has contexts Ξ as a parameter, by changing Ξ we are able to model different types of interaction
- This definition of refinement can be further specialised for refinement of specific cases

VERTICAL REFINEMENT

- We can view each special model of refinement as a layer in the grand scheme of things each encompassing a set of entities and a refinement relation
- Mathematically our vertical refinement is a Galois connection between the layers.
- This allows us to interpret high level entities as low level entities using a semantic mapping, however, these low level entities cannot interact with the high level ones so the contexts must also be refined

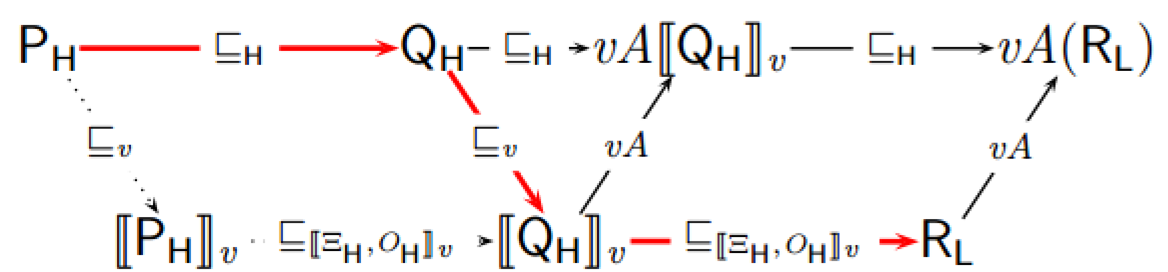


Fig Refinement ⟶ within and between layers

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