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IRISH RESEARCH COUNCIL
An Chomhairle um Thaighde in Éirinn

EXAMINING REFINEMENT: THEORY, TOOLS AND MATHEMATICS

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PROBLEM

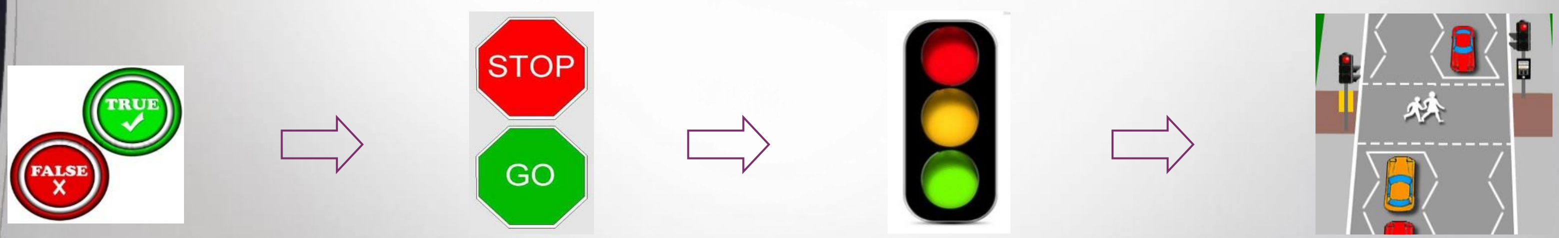
❖ Different formalisms do not integrate well e.g. Event B only models the specification and its proofs are not easily transferable to other formalisms

PROPOSED SOLUTION

- ❖ Establish a theoretical framework within which refinement steps, and their associated proof obligations, can be shared between different formalisms
- ❖ Hypothesis: the theory of institutions can provide this framework and, we will construct an institution based specification of the Event B formalism

REFINEMENT

❖ In software engineering it is common to model systems at different levels of abstraction



❖ We can map between these different levels of abstraction in a verifiable way through a process known as refinement

REDUCING NONDETERMINISM

$$\begin{aligned}&if\ a \le b \to a := a - b\\ &\square\ b \ge a \to b := b - a\\ &fi\end{aligned}$$

$$\sqsubseteq$$

$$\begin{aligned}&if\ a \le b \to a := a - b\\ &\square\ a \nleq b \to b := b - a\\ &fi\end{aligned}$$

This one is nondeterministic when a=b

This one is deterministic

Classic example: Converting an NFA to a DFA

THEORIES OF REFINEMENT

- Main theories developed by Carroll Morgan, Ralph Johan Back and Joseph Morris
- All three are based on Dijkstra's language of guarded commands and weakest precondition calculus.
- Morgan takes a very program oriented view whereas Back appears to be much more theoretical with foundations in lattice and category theory. Morris extended Back's work with prescriptions.

MORGAN'S REFINEMENT

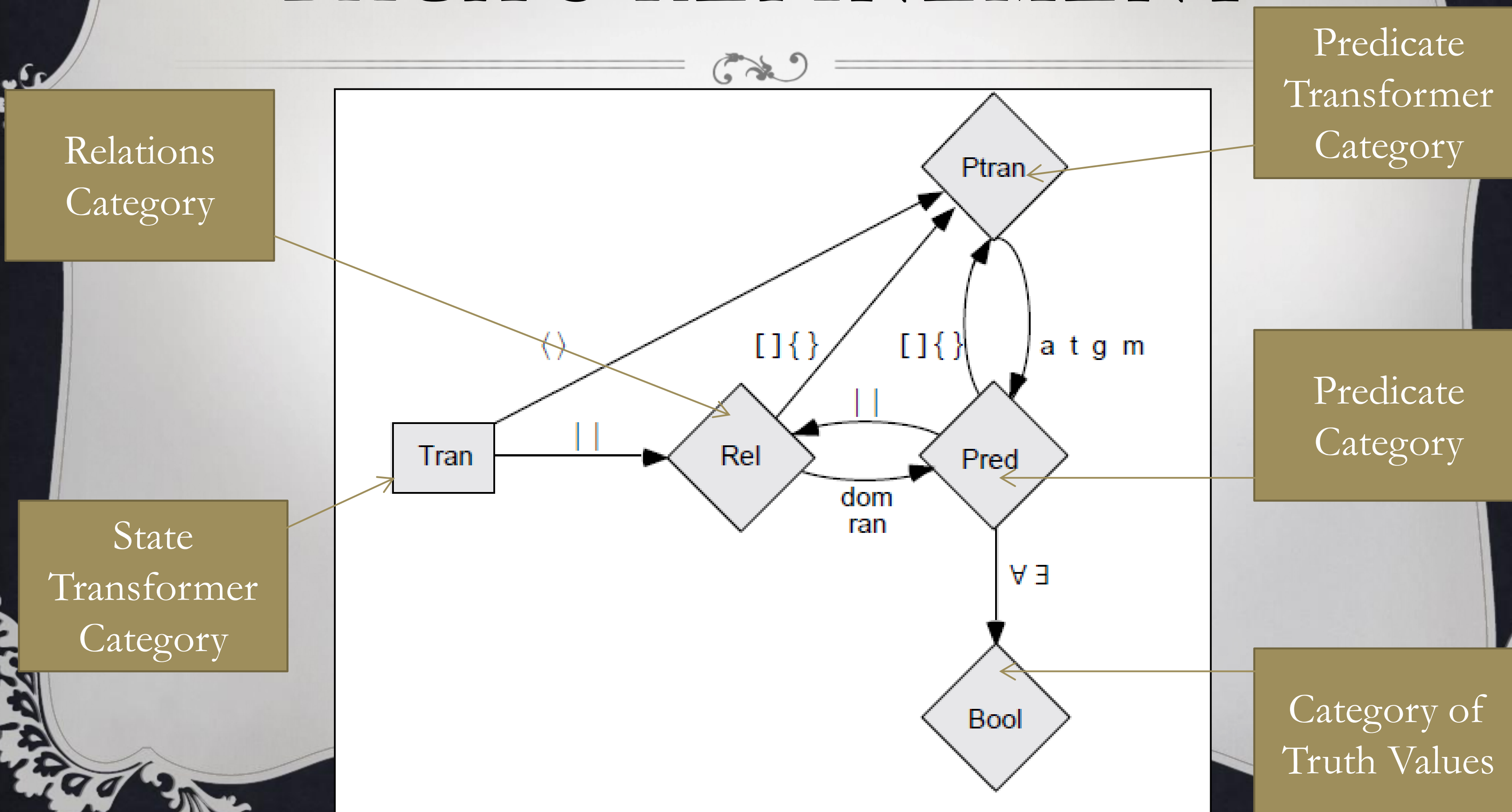
- Weakening the precondition
- Strengthening the postcondition
- Introducing local variables
- Renaming local variables
- Introducing logical constants
- Eliminating logical constants
- Expanding the frame
- Introducing `skip`
- Introducing `abort`
- Introducing assignment
- Introducing sequential composition
- Introducing alternation
- Introducing iteration

MORGAN'S REFINEMENT

Introducing alternation

$$w : [pre \land (\bigvee i \bullet G_i), post] = \texttt{if} \;\; (\Box i \bullet G_i \to w : [pre \land G_i, post]) \;\; \texttt{fi}$$

BACK'S REFINEMENT



BACK'S REFINEMENT

- Similar rules to Morgan's refinement calculus
- Example
  - Introduce conditional :

$$S \sqsubseteq [g_1 \cup \cdots \cup g_n] \,;\, \text{if } g_1 \rightarrow S \,[]\, \cdots \,[]\, g_n \rightarrow S \text{ fi} \,.$$

MORRIS REFINEMENT

- Extended Back's calculus with prescriptions
- A prescription $P||Q$ specifies a mechanism that when executed in a state satisfying P will terminate in a state satisfying Q
  - P and Q are predicates

$$P||Q \sqsubseteq s_1 \sqsubseteq s_2 \sqsubseteq \cdots \sqsubseteq s$$

MORRIS REFINEMENT

- Given $P||Q$ there are 6 ways of choosing s such that

$$P||Q \sqsubseteq s$$

1. Skip
2. Assignment
3. Prescription
4. If statement
5. Composition
6. Block

$$P||Q \sqsubseteq R||S;T||U \ if \ \lceil P \Rightarrow R \rceil, \lceil S \Rightarrow T \rceil$$
$$and \ \lceil U \Rightarrow Q \rceil$$

GENERAL REFINEMENT

"The abstract entity **A** is refined by the concrete entity **C** if no user of **A** could observe if they were given **C** in its place"

Liskov Substitution

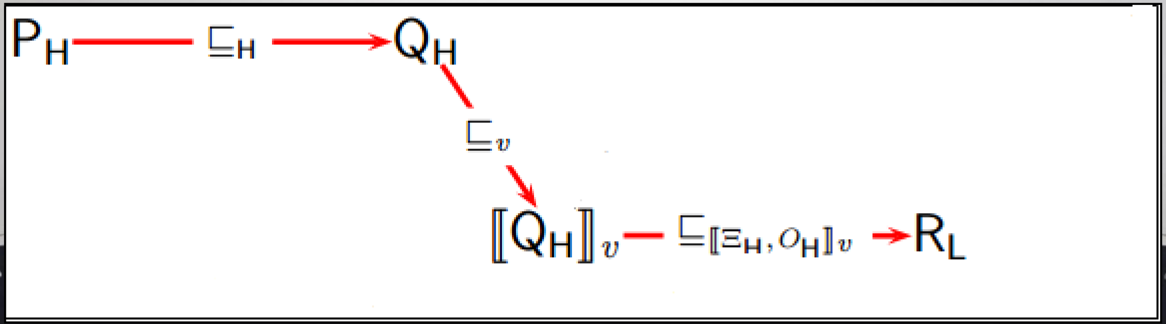
GENERAL REFINEMENT

- 3 main components:
  1. Set of entities – specifications and implementations
  2. Set of contexts – the environment with which the entities interact
  3. A user formalised by defining the set of observations that can be made when an entity is executed in a given context
- Example: an entity as a motor, a context as the car in which the motor runs and the user as the driver of the car

$$\mathsf{A} \sqsubseteq_{\Xi,O} \mathsf{C} \triangleq \forall x \in \Xi . O([\mathsf{C}]_x) \subseteq O([\mathsf{A}]_x)$$

SPECIAL THEORIES

- We can view each special model of refinement as a layer in the grand scheme of things each encompassing a set of entities and a refinement relation
- This allows us to interpret high level entities as low level entities using a semantic mapping, however, these low level entities cannot interact with the high level ones so the contexts must also be refined



GALOIS CONNECTIONS

❖ Mathematically this vertical refinement is a Galois connection between the layers.

❖ Given two posets $(A, \leq_A)$ and $(B, \leq_B)$. A Galois connection between these posets consists of two maps $f: A \rightarrow B$ and $g: B \rightarrow A$, such that for all $a \in A$ and $b \in B$, we have

- $a \leq_A f(g(a))$
- $f(g(b)) \leq_B b$

Π -INSTITUTIONS

- Alternative to institution – replacing the notions of model and satisfaction by Tarski's consequence operator
- Definition:
  - A π-institution is a triple (Sign, φ, $\{Cn_\Sigma\}_{\Sigma:Sign}$) consisting of
    1. A category Sign (of signatures)
    2. A functor φ:Sign -> Set (set of formulae over each signature)
    3. For each object Σ of Sign, a consequence operator Cn_Σ defined in the power set of φ(Σ) satisfying for each A, B ⊆ φ(Σ) and μ: Σ -> Σ

(RQ1) $A \subseteq Cn_\Sigma(A)$ (Extensiveness)

(RQ2) $Cn_\Sigma(Cn_\Sigma(A)) = Cn_\Sigma(A)$ (Idempotence)

(RQ3) $Cn_\Sigma(A) = \bigcup_{B\subseteq A, B\ finite} Cn_\Sigma(B)$ (Compactness)

(RQ4) $\varphi(\mu)(Cn_\Sigma(A)) \subseteq Cn_{\Sigma'}(\varphi(\mu)(A))$ (Structurality)

TARSKI'S CONSEQUENCE OPERATOR

- Axiom 1:

$$|S| \leq \aleph_0$$

- Axiom 2:

$$If\ X \subseteq S, then\ X \subseteq Cn(X) \subseteq S$$

- Axiom 3:

$$If\ X \subseteq S, then\ Cn\big(Cn(X)\big) = Cn(X)$$

- Axiom 4:

$$If\ X \subseteq S, then\ Cn(X) = \sum_{Y \subseteq X\ and\ |Y| < \aleph_0} Cn(Y)$$

- Axiom 5:

$$\exists x \in S\ such\ that\ Cn(\{x\}) = S$$

$$f: \mathrm{A} \rightarrow \mathrm{B}$$

$$f(x) = Cn(x)$$

$$g: \mathrm{B} \rightarrow \mathrm{A}$$

$$g(\mathrm{x}) = \bigcap_{Y \subseteq S \text{ and } X \subseteq Cn(Y)} Y$$

Both posets are ordered by set theoretic inclusion

EVENT B

- The Event B formal specification language is used in the verification of safety critical systems

- Event B models are an instance of the specification

Machine	Context
variables	carrier sets
invariants	constants
events	axioms

```
MACHINE
  mac1
VARIABLES
  cars_go
  peds_go
INVARIANTS
  inv1  :  cars_go ∈ BOOL
  inv2  :  peds_go ∈ BOOL
  inv3  :  ¬(peds_go=TRUE ∧ cars_go=TRUE)
EVENTS
  INITIALISATION  ≙
      STATUS
   ordinary
  BEGIN
   act1  :  cars_go ≔ FALSE
   act2  :  peds_go ≔ FALSE
  END

  set_peds_go  ≙
      STATUS
   ordinary
  WHEN
   grd1  :  cars_go = FALSE
  THEN
   act1  :  peds_go ≔ TRUE
  END

  set_peds_stop  ≙
      STATUS
   ordinary
  BEGIN
   act1  :  peds_go ≔ FALSE
```

Gluing Invariant

```
MACHINE
  mac2
REFINES
  mac1
SEES
  ctx1
VARIABLES
  cars_colour
  peds_colour
INVARIANTS
  inv1  :  peds_colour ∈ {red, green}
  inv2  :  peds_go = TRUE ⇔ peds_colour = green
  inv3  :  cars_colour ∈ {red, green}
  inv4  :  cars_go = TRUE ⇔ cars_colour = green
EVENTS
  INITIALISATION  ≙
      STATUS
   ordinary
  BEGIN
   act1  :  cars_colour ≔ red
   act3  :  peds_colour ≔ red
  END

  set_peds_green  ≙
      STATUS
   ordinary
  REFINES
   set_peds_go
  WHEN
   grd1  :  cars_colour = red
  THEN
   act1  :  peds_colour ≔ green
  END

  set_peds_red  ≙
      STATUS
   ordinary
  REFINES
   set_peds_stop
  BEGIN
   act1  :  peds_colour ≔ red
  END
```

JML

- JML = Java Modelling Language
- Specifications are annotations:

```
/*@ requires array.length>0;
    ensures sorted(array);
 @*/
public int [] sort(int [] array){

    int temp =0;

    for(int j=0;j<array.length-1;j++){

        if(array[j]>array[j+1]){
            temp = array[j];
            array[j] = array[j+1];
            array[j+1] = temp;
        }
    }
    return array;
}
```

```
/*@ requires a.length>0;
    assignable \nothing;
@*/
public boolean sorted(int [] a)
{
    boolean valid = true;
    for(int i=0;i<a.length-1;i++)
    {
        if(a[i]>a[i+1])
        {
            valid = false;
            break;
        }
    }
    return valid;
}
```

REFINEMENT IN JML

❖ JML supports refinement as specification inheritance

```
//@ public model non_null String name;
private /*@ non_null @*/ String fullName;
//@ private represents name <- fullName;
```

```
package org.jmlspecs.samples.jmltutorial;

//@ refine "Person.java";

public class Person {
  private /*@ spec_public non_null @*/
    String name;
  private /*@ spec_public @*/
    int weight;

  /*@ public invariant !name.equals("")
    @         && weight >= 0; @*/

  //@ also
  //@ ensures \result != null;
  public String toString();

  //@ also
  //@ ensures \result == weight;
  public /*@ pure @*/ int getWeight();

  /*@ also
    @ requires kgs >= 0;
    @ requires weight + kgs >= 0;
    @ ensures weight == \old(weight + kgs);
    @*/
  public void addKgs(int kgs);

  /*@ also
    @ requires n != null && !n.equals("");
    @ ensures n.equals(name)
    @   && weight == 0; @*/
  public Person(String n);
}
```

AIM

- ❖ Establish a theoretical framework within which refinement steps, and their associated proof obligations, can be shared between different formalisms

FUTURE WORK

1. Specify a π -institution for refinement in at least two formalisms
2. Complete refinement case studies in both formalisms
3. Use π-institutions to combine proofs in these formalisms