COMMUNICATING SEQUENTIAL PROCESSES

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PROCESSES

- The behaviour pattern of an object described in terms of the events in its alphabet.
- Example:
  - A counter starts on the bottom left square of a board and can only move up or right to a white square
  - $\alpha CTR = \{up, right\}$
  - $CTR = \{right \rightarrow up \rightarrow right \rightarrow right \rightarrow STOP_{\alpha CTR}\}$

RECURSION

- Notation: $X = F(X)$ becomes $\mu X: A.F(X)$
  - Where X is the bound variable and A is the alphabet

- Example: A perpetual clock
  - $CLOCK = \mu X: \{tick\}.(tick \rightarrow X)$

CHOICE

- If x and y are distinct choices and P and Q processes then
  - $(x \to P | y \to Q)$
- Example:
  - A vending machine which offers a choice of input coins and a choice of either a small or large biscuit and change
  - $VMC = (in2p \to (large \to VMC \,|small \to out1p \to VMC) \,|in1p \to (small \to VMC \,|in1p \to (large \to VMC \,|in1p \to STOP)))$

IMPLEMENTATION OF PROCESSES

- Every process can be written in the form $(x: B \rightarrow F(x))$
  - The process may be regarded as a function F.
  - With a domain B defining the set of events in which the process is initially prepared to engage.
  - For each x in B, F(x) defines the future behaviour of the process if the first event was x.

LISP

❖ Each event is an atom

❖ A process is a function which can be applied to a symbol

❖ If the symbol is not a possible first event for the process then the function returns "BLEEP

❖ Example: Binary choice $(c \rightarrow P|d \ \rightarrow Q)$

- $choice2(c, P, d, Q) = \lambda x. if\ x = c\ then\ P$
  $else\ if\ x = d\ then\ Q$
  $else\ "BLEEP$

TRACES

❖ A trace of the behaviour of a process is a finite sequence of symbols recording the events in which the process has engaged up to some moment in time.

❖ Example: A trace of a vending machine at the moment it is finished serving its first two customers

- <coin, choc, coin, choc>

OPERATIONS ON TRACES

1. Catenation $< coin, choc > \wedge < coin, toffee > =< coin, choc, coin, toffee >$
2. Restriction $< around, up, down, around > \upharpoonright \{up, down\} =< up, down >$
3. Head and Tail $< x, y, x >_0 = \mathrm{x}$ $\quad < \mathrm{x}, \mathrm{y}, \mathrm{x} >' = < \mathrm{y}, \mathrm{x} >$
4. Star $A^* = \{s | s \upharpoonright A = s\}$
5. Ordering $s \leq t = (\exists u. s \wedge u = t$
6. Length $\# < \mathrm{x}, \mathrm{y}, \mathrm{x} >= 3$

IMPLEMENTATION OF TRACES

- Traces are implemented by lists of atoms representing their events
- Operations on traces can be readily implemented as functions on lists
  - $s^\wedge t = append(s, t)$
- Example :Restriction
  - $isMember(x, B) = if\ B = NIL\ then\ false$
    $else\ if\ x = car(B) then\ true$
    $else\ isMember(x, cdr(B))$
  - $restrict(s, B) = if\ s = NIL\ then\ NIL$
    $else\ if\ isMember(car(s), B)$
    $then\ cons\big(car(s), restrict(cdr(s), B)\big)$
    $else\ restrict(cdr(s), B)$

TRACES OF A PROCESS

❖ Before a process starts it is not known which trace will occur the choice depends on environmental factors beyond the control of the process

❖ However we can know the complete set of possible traces and this is denoted as the function traces(P)

❖ Example: A perpetual clock

- $traces(\mu X.tick \rightarrow X) = \{<>, <tick>, <tick, tick>, \ldots\}$
  $= \{tick\}^*$

IMPLEMENTATION

- $isTrace(s, P) = if\ s = NIL\ then\ true$

  $else\ if\ P(s_0) =$ "BLEEP then false

  $else\ isTrace\ (s', P(s_0))$

AFTER

- $If\ \ s \in traces(P)$

$$then\ P/s$$

Is a process that behaves the same as P behaves from the time after P has engaged in all the actions recorded in the trace s

SPECIFICATIONS

- In the case of a process the most relevant observation of behaviour is the trace of events that occur up to a given moment in time.
- Use the variable tr to stand for an arbitrary trace of the process being specified.
- Use $tr \downarrow c = \#(tr \upharpoonright \{c\})$
  - i.e. the number of occurrences of the symbol c in tr

EXAMPLE

❖ The customer of a vending machine wants to ensure that it will not absorb further coins until it has dispensed the chocolate already paid for

- $FAIR1 = ((tr \downarrow coin) \leq (tr \downarrow choc) + 1)$

SATISFACTION

❖ If P is a product which meets a specification S we say that *P satisfies S*.

- P sat S
- $\forall tr.\, tr \in traces(P) \Rightarrow S$

QUESTIONS?