Self-Simulation in the Collatz Process

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Abstract

The Collatz problem studies the iteration of the following operator defined on non-negative integers:

\[ T(n) = \begin{cases} 
T_0(n) = n/2 & n \equiv 0 \mod 2 \\
T_1(n) = (3n + 1)/2 & n \equiv 1 \mod 2 
\end{cases} \]

The directed graph \( G \) associated to the operator \( T \) is such that its nodes are non-negative integers and that there exists an arrow between \( x \) and \( y \) if and only if \( y = T(x) \). By understanding the structure of paths in this graph (see [2]), we design a method to highlight self-simulation in the Collatz process: we construct numbers whose Collatz sequence simulates the Collatz sequence of other numbers. This method is rooted in number theoretic considerations and highlights the role of the parity of the sequence \( (2^{-1})^m \mod 3 \) where \( 2^{-1} \) is the modular inverse of 2 in \( \mathbb{Z}/3\mathbb{Z} \) and \( m \) ranges over non-negative integers. Enventually, we can connect this approach to a much more algorithmic one which studies the Collatz process as an algorithm on binary strings (see [1]).

As a result of this work, we get a better understanding of paths in the Collatz graph and how these paths can produce a self-simulating behavior.

References


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