Augmenting The Theory Of Changes

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Abstract

Automatic differentiation (AD for short) is an invaluable technique within numerical computing, making derivative calculation accurate, easy, and efficient. AD comes in two so-called "modes", forward and reverse mode. With forward mode propagating derivatives forward through the computation, and reverse mode, as the name suggests, propagating derivatives backward. The idea of derivatives has taken hold in other areas of computer science. Once sucg area is the incremental lambda calculus, which provides a formal framework for incremental computation through a "theory of changes". Incremental computation is the pursuit of efficient computation by avoiding re-computation of functions for small changes in parameters. This *incrementalisation* in the formal system is achieved through program *derivatives*. These derivatives are a map from a change in input to a change in output of a function, much like mathematical ones. However they are discrete valued and thus have more attractive properties. The functions in question can be an arbitrary lambda calculus expression, and only needs what the author calls a *change structure* to calculate. We suggest an augmentation in two orthogonal directions with the intent to prove some properties of automatic differentiation, in particular - forward mode.