



NUI MAYNOOTH

Ollscoil na hÉireann Má Nuad



JANUARY 2010 EXAMINATION

CS605

The Mathematics and Theory of Computer Science

Dr. L. Rapanotti, Dr. A. Winstanley, Mr. T. Naughton

Time allowed: 3 hours

Answer **three** questions

All questions carry equal marks

Additional material allowed:

One copy of M. Sipser, *Introduction to the Theory of Computation* (PWS, Boston, 1997), containing no annotations or extra pages AND/OR one copy of M. Sipser, *Introduction to the Theory of Computation – Second Edition* (Thomson, Boston, 2005), containing no annotations or extra pages. Please see declaration page at back.

- 1
 - (a) Prove that the regular languages are closed under intersection, i.e. that the intersection of two regular languages is a regular language. **[10 marks]**
 - (b) Let $L = \{ \langle M \rangle : M \text{ is a TM with an input alphabet of } \{a,b\} \text{ and } M \text{ accepts no words} \}$. Prove that the complement of L is Turing-recognisable. **[10 marks]**
 - (c) How can we use a reduction to prove nonmembership of a class? **[5 marks]**

- 2
 - (a) For each of the following languages, prove that it is regular or prove that it is not regular. Note, the marks are not divided equally between each part (more marks will be given for proving a language is not regular than for proving a language is regular).
 - i. $\{uv : u,v \in \{0,1\}^*, u \text{ contains the same number of 1s as } v\}$
 - ii. $\{uxv : u,v \in \{0,1\}^*, |u| = |v|\}$
 - iii. $\{uv : u,v \in \{0,1\}^*, |u| = |v|\}$
 - (b)
 - i. Prove that $L = \{uxv : u,v \in \{a,b\}^*, |u| = |v|, u \neq v^R\}$ is a context-free language. **[8 marks]**
 - ii. Can a deterministic PDA accept L ? Justify your answer. **[2 marks]**

- 3
 - (a) Prove that the set of context-free languages is not equal to the set of decidable languages. You can choose a non-context-free language of your choice without having to prove that it is not context-free. **[5 marks]**
 - (b) Ireland's largest supermarket chain decides to lower the price of a number of selected brands of bottled water and wants every one of its stores to participate. Unfortunately, while the larger stores sell each of the m brands, some of the smaller stores sell only a limited range. The supermarket chain wants to discount no more than k possible brands such that each of its n stores stocks at least one discounted brand.
 - i. List the steps required to prove any given problem is NP-complete. **[5 marks]**
 - ii. Prove that this particular problem above is NP-complete. You are given only that the problem 3-SAT is NP-complete. **[15 marks]**

- 4 (a) Let machine N be defined as $N = \{Q, \Sigma, \delta, q_0, F\}$, where $Q = \{99, 01\}$, $\Sigma = \{a, b\}$, $q_0 = 99$, $F = \{99\}$, and δ is given by the following table. [5 marks]

$Q \backslash \Sigma$	a	b	ϵ
99	{99}	{01}	{01}
01	{01}	{01}	\emptyset

- i. Convert the NFA N into an equivalent FA. You may use any technique you wish.
 - ii. Convert the NFA N into an equivalent NFA with a single state.
- (b) For each of the following languages, prove that it is decidable or prove that it is undecidable. Note, the marks are not divided equally between each part (more marks will be given for proving a language is undecidable than for proving a language is decidable). If appropriate, you may answer this question by assigning a name, mathematical construct, or piece of pseudocode to each of the numbered blanks in the proof template in Figure 1 on page 3. Where blanks have the same number, this denotes their contents will be the same. Alternatively, if appropriate, you can choose to ignore the template and construct your own proof from scratch. You are given that A_{TM} is undecidable. A_{TM} is defined as $A_{TM} = \{ \langle M, w \rangle : M \text{ is a TM and } M \text{ halts on input } w \}$. [20 marks]
- i. $EW_{TM} = \{ \langle M \rangle : M \text{ is Turing machine that halts if given the empty word as input} \}$
 - ii. $STATENOENTER_{TM} = \{ \langle M, q \rangle : M \text{ is a TM, } q \text{ is a state in } M, \text{ and } M \text{ never goes into state } q \text{ when it is run} \}$
 - iii. $RESPONSE_{BIN} = \{ \langle P, f, t \rangle : P \text{ is a program written in machine code for Industrial Automation's R2 series ELIW processor, } f \text{ is the clock frequency of the processor, } t \text{ is a number of seconds, and when } P \text{ is executed on an R2 processor with speed } f \text{ it halts within } t \text{ seconds} \}$

Proof. We will use a mapping reduction to prove the reduction 1. Assume that 2 is decidable. The function f that maps instances of 3 to instances of 4 is performed by TM F given by the following pseudocode.

$F =$ “On input $\langle \underline{5} \rangle$:

1. Construct the following M' given by the following pseudocode.

$M' =$ “ 6 ”

2. Output $\langle \underline{7} \rangle$ ”

Now, $\langle \underline{7} \rangle$ is an element of 8 iff $\langle \underline{5} \rangle$ is an element of 9. So using f and the assumption that 2 is decidable, we can decide 10. A contradiction. Therefore, 2 is undecidable. (This also means that the complement of 2 is undecidable; the complement of any undecidable language is itself undecidable.)

Figure 1. Proof template that may be appropriate for question 4 on page 2.



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Declaration

To be signed by the student and collected by an
invigilator at the beginning of the examination

1. I have searched through my copies of M. Sipser, *Introduction to the Theory of Computation*, first and second editions, (the Sipser book) and it does not contain any extra pages or annotations (except for annotations that correct minor typographical errors).
2. I understand that by failing to notify an invigilator of any annotations or extra pages in my copies of the Sipser book, I will receive a mark of zero in this examination. This does not affect any further disciplinary actions that the University may wish to take.
3. I understand also that directly copying large amounts of material from the Sipser books without substantially tailoring it to the question asked may result in a mark of zero.

Print name _____ Student number _____

Signed _____ Date _____