OLLSCOIL NA hÉIREANN, MÁ NUAD

NATIONAL UNIVERSITY OF IRELAND, MAYNOOTH

M.SC. IN SOFTWARE ENGINEERING EXAMINATION

SAMPLE 2003-2004

PAPER CS605

MATHEMATICS AND THEORY OF COMPUTER SCIENCE

Dr. Andrew Martin, Prof. R. Reilly, Mr. T. Naughton.

Attempt any THREE questions. Time Allowed: 3 hours.
Additional material allowed: One copy of M. Sipser, *Introduction to the Theory of Computation* (PWS, Boston 1997), containing no annotations or extra pages.

1. (a) For each of the following languages, prove that it is regular or prove that it is not regular. [15 marks]
   i. $L_0 = \{uv : u, v \in \{0,1\}^*, u \text{ contains the same number of 1s as } v\}$
   ii. $L_1 = \{uxv : u, v \in \{0,1\}^*, |u| = |v|\}$
   iii. $L_2 = \{uv : u, v \in \{0,1\}^*, |u| = |v|\}$

   (b) Let $A = \{a : a \in \mathbb{R}, 0 \leq a < 1\}$. Prove that A is uncountable. [10 marks]

2. (a) Prove that the set of rational numbers is countable. The set of rational numbers $\mathbb{Q}$ is defined as $\mathbb{Q} = \{\frac{m}{n} : m \in \mathbb{N}, n \in \mathbb{Z}\}$. [5 marks]

   (b) i. Prove that $L = \{uxv : u, v \in \{a,b\}^*, |u| = |v|, u \neq v^R\}$ is a context-free language. [8 marks]
   ii. Can a deterministic PDA accept L? Justify your answer. [2 marks]

   (c) Prove that the set of context-free languages is not equal to the set of decidable languages. [5 marks]

   (d) How can we use a reduction to prove nonmembership of a class? [5 marks]

Proof. We will use a mapping reduction to prove the reduction _____. Assume that _____ is decidable. The function f that maps instances of _____ to instances of _____ is performed by TM F given by the following pseudocode.

$F =$ "On input $\langle$_____$\rangle$:
1. Construct the following M' given by the following pseudocode.
   $M' =$ "_____"
2. Output $\langle$_____$\rangle$"

Now, $\langle$_____$\rangle$ is an element of _____ iff $\langle$_____$\rangle$ is an element of _____. So using f and the assumption that _____ is decidable, we can decide _____. A contradiction. Therefore, _____ is undecidable. (This also means that the complement of _____ is undecidable; the complement of any undecidable language is itself undecidable.)

Figure 1: Proof template for question 4b.

3. (a) Rewrite the function $f : A \rightarrow 2^B$ as a cross product. [5 marks]

   (b) Let $L = \{\langle M \rangle : M$ is a TM with an input alphabet of $\{a, b\}$ and M accepts no words$\}$. Prove that $\overline{L}$, the complement of L, is Turing-recognisable. [10 marks]

   (c) Convert the NFA N into a FA. N is defined as $N = \{Q, \Sigma, \delta, q_0, F\}$, where $Q = \{99, 01\}$, $\Sigma = \{a, b\}$, $q_0 = 99$, $F = \{99\}$, and δ is given by the following table. [5 marks]

Q\Σ	a	b	e
99	{99}	{01}	{01}
01	{01}	{01}	{01}

   (d) It has been argued that writing programs for NFAs is a waste of time because a NFA could never be built. Comment. [5 marks]

4. (a) You are given that the Hitting Set problem is in $\mathcal{NP}$. Prove that it is $\mathcal{NP}$-complete. [5 marks]

   (b) For each of the following languages, prove that it is decidable or prove that it is undecidable. You may use the proof template in Figure 1 if you wish, and if it is necessary. You are given that A_{TM} is undecidable. A_{TM} is defined as $A_{TM} = \{\langle M, w \rangle : M$ is a TM and M halts on input $w\}$. [20 marks]

      i. $H_{TM} = \{\langle M \rangle : M$ is a halting TM$\}$
      ii. $STATENOENTER_{TM} = \{\langle M, q \rangle : M$ is a TM, q is a state in M, and M never goes into state q when it is run$\}$
      iii. $RESPONSE_{BIN} = \{\langle P, f, t \rangle : P$ is a program written in machine code for Industrial Automation's R2 series ELIW processor, f is the speed of the processor (number of instruction words per second), t is a number of seconds, and when P is executed on an R2 processor with speed f it halts within t seconds$\}$

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Declaration

To be signed by the student and collected by an invigilator at the beginning of the examination

1. I have searched through my copy of M. Sipser, *Introduction to the Theory of Computation*, PWS, Boston 1997 (the Sipser book) and it does not contain any extra pages or annotations (except for annotations that correct minor typographical errors).

2. I understand that by failing to notify an invigilator of any annotations or extra pages in my copy of the Sipser book, I will receive a mark of zero in this examination. This does not affect any further disciplinary actions that the University may wish to take.

3. I understand also that directly copying large amounts of material from the Sipser book without substantially tailoring it to the question asked may result in a mark of zero.

Print name _____ Student number _____

Signed _____ Date _____