OLLSCOIL NA hÉIREANN, MÁ NUAD

NATIONAL UNIVERSITY OF IRELAND, MAYNOOTH

M.SC. IN SOFTWARE ENGINEERING EXAMINATION

DECEMBER 2003

PAPER CS605

MATHEMATICS AND THEORY OF COMPUTER SCIENCE

Dr. Andrew Martin, Prof. R. Reilly, Mr. T. Naughton.

Attempt any THREE questions. Time Allowed: 3 hours.
Additional material allowed: One copy of M. Sipser, *Introduction to the Theory of Computation* (PWS, Boston 1997), containing no annotations or extra pages.

1. For each of the following languages, prove that it is regular or prove that it is not regular. [25 marks]
   (a) $L_0 = \{uv : u \in \{0,1\}^*, v \in \{0\}^*, |u| = |v|\}$
   (b) $L_1 = \{uv : u \in \{0,1\}^*, v \in \{0,1\}^*, |u| = |v|\}$
   (c) $L_2 = \{w : w \in \{0,1\}^*, |w|$ is odd and the middle symbol of w is $0\}$
   (d) $L_3 = \emptyset$
   (e) $L_4 = \{ww^{\mathrm{R}} : w \in \{0\}^*\}$

2. (a) Construct a FA that accepts the language $L = \{w : w \in \{a,b\}^*, w$ contains the substring baa but does not contain it at the beginning of the word$\}$. [5 marks]
   (b) Let the language $\text{VARNEG}_{\text{C++}}$ be defined as $\text{VARNEG}_{\text{C++}} = \{\langle C, v\rangle : C$ is a C++ program, v is an integer variable declared in C, and the value in v goes negative at least once in the execution of $C\}$. Prove that $\text{VARNEG}_{\text{C++}}$ is undecidable. You may use the proof template in Figure 1 if you wish. You are given that $\text{HALT}_{\text{C++}}$ is undecidable. $\text{HALT}_{\text{C++}}$ is defined as $\text{HALT}_{\text{C++}} = \{\langle C, w\rangle : C$ is a C++ function, and C halts on its string input $w\}$. [10 marks]
   (c) Prove that $\text{VARNEG}_{\text{C++}}$ is Turing recognisable or prove that it is not Turing recognisable. [5 marks]
   (d) Give a definition of the language $\overline{\text{VARNEG}_{\text{C++}}}$ (the complement of $\text{VARNEG}_{\text{C++}}$). Prove that $\overline{\text{VARNEG}_{\text{C++}}}$ is Turing recognisable or prove that it is not Turing recognisable. [5 marks]

Proof. We will use a mapping reduction to prove the reduction _____. Assume that _____ is decidable. The function f that maps instances of _____ to instances of _____ is performed by TM F given by the following pseudocode.

$F =$ "On input $\langle$_____$\rangle$:
1. Construct the following M' given by the following pseudocode.
   $M' =$ "_____"
2. Output $\langle$_____$\rangle$"

Now, $\langle$_____$\rangle$ is an element of _____ iff $\langle$_____$\rangle$ is an element of _____. So using f and the assumption that _____ is decidable, we can decide _____. A contradiction. Therefore, _____ is undecidable. (This also means that the complement of _____ is undecidable; the complement of any undecidable language is itself undecidable.)

Figure 1: Proof template for question 2b.

3. (a) Prove that $L = \{wxu : w, u \in \{a, b\}^*, u^{\mathrm{R}}$ is a substring of $w\}$ is a context-free language. [8 marks]

   (b) Can a deterministic PDA accept L? Justify your answer. [2 marks]

   (c) Give an outline of a proof that the set of context-free languages is a proper subset of the set of decidable languages. [5 marks]

   (d) Let the countable set S be the set of all subsets of the set A. Argue why A must be finite. [5 marks]

   (e) Prove that the set of regular languages is closed under complement. The complement of a language L over alphabet Σ is defined as $\overline{L} = \{w : w \in \Sigma^*, w \notin L\}$. [5 marks]

4. (a) Why do we use languages to study the power of computing devices? [5 marks]

   (b) Construct a TM M, with as many tapes as you like, that recognises the language $L = \{a^n b^{2n} : n \geq 0\}$. Clearly indicate the start state and accepting state(s). [5 marks]

   (c) Construct a TM M' that decides L. You do not need to write out the full table of behaviour for M', just specify how M needs to be modified. If you create new states, explain their purpose. [5 marks]

   (d) Let $L = \{w : w \in \{a, b\}^*, w$ does not contain the substring $aa\}$. Let $X = L \times \mathbb{Z}$. Prove that X is countable. [5 marks]

   (e) Let $\text{HITME}_{\text{C++}} = \{\langle\{A_0, A_1, \ldots, A_{m-1}\}, k\rangle$: each $A_i \subset \mathbb{N}$ is a finite set, $k \in \mathbb{N}$, and a set $B \subset \mathbb{N}$ exists with $|B| \leq k$ that has a nonempty intersection with each $A_i\}$. Prove that $\text{HITME}_{\text{C++}}$ is in $\mathcal{NP}$. [5 marks]

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Declaration

To be signed by the student and collected by an invigilator at the beginning of the examination

1. I have searched through my copy of M. Sipser, *Introduction to the Theory of Computation*, PWS, Boston 1997 (the Sipser book) and it does not contain any extra pages or annotations (except for annotations that correct minor typographical errors).

2. I understand that by failing to notify an invigilator of any annotations or extra pages in my copy of the Sipser book, I will receive a mark of zero in this examination. This does not affect any further disciplinary actions that the University may wish to take.

3. I understand also that directly copying large amounts of material from the Sipser book without substantially tailoring it to the question asked may result in a mark of zero.

Print name _____ Student number _____

Signed _____ Date _____