OLLSCOIL NA hÉIREANN, MÁ NUAD

NATIONAL UNIVERSITY OF IRELAND, MAYNOOTH

M.SC. IN SOFTWARE ENGINEERING EXAMINATION

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COMPUTER SCIENCE

PAPER CS605

MATHEMATICS AND THEORY OF COMPUTER SCIENCE

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Answer three (3) questions from four (4). Time Allowed: 3 hours.
Additional material allowed: One copy of M. Sipser: *Introduction to the Theory of Computation* (PWS, Boston 1997), containing no annotations or extra pages.

1. (a) Expand the languages defined by the following expressions. Note, e denotes the empty word, $\circ$ denotes concatenation, $\emptyset$ denotes the empty set, and 2^L denotes the power set of L. [6 marks]
   i. $\emptyset \cup \{aa, ab\}$
   ii. $\{e\}^*$
   iii. $\emptyset^*$
   iv. $\emptyset \circ \{a, b, c\}$
   v. 2^L, where the language $L = \{e, ab\}$
   vi. the regular expression $(0 \cup e)1$

   (b) Prove that the regular languages are closed under concatenation. [6 marks]

   (c) Can you enumerate the set of all words over a finite alphabet? Prove your answer. [5 marks]

   (d) Explain the following properties of languages: *acceptable*, *decidable*, *recursively enumerable*, and *recursive*. Give an example in each case. [8 marks]

2. Define a language that is recursively-enumerable and nonrecursive. Prove that your language has both properties. Full marks will be awarded for an unambiguous definition and a detailed proof. [25 marks]

3. (a) For each of the following languages, prove that it is regular or prove that it is not regular. [13 marks]

   i. $\{w : w \in \{a, b\}^*, w$ is the empty word or contains the substring $aab\}$

   ii. $\{ww : w \in \{a, b\}^*\}$

   iii. $\{uv : u, v \in \{a, b\}^*, u$ is not equal to $v\}$

   (b) Prove that the set of regular languages is a proper subset of the set of the context-free languages. The only theorems you may use (if you wish to) are those you have proved from part (a) of this question and the following [12 marks]

   - a language is regular iff it is accepted by a finite automaton
   - a language is context-free iff it is accepted by a pushdown automaton.

4. (a) Define any decision problem relating to finite automata. State whether this problem would be decidable or not by a Turing machine. Prove your answer for full marks. [8 marks]

   (b) What does a reduction $A \leq B$ between two problems A and B establish about the relative computability of A and B? What does a polynomial reduction establish about the relative computational complexity of A and B? [2 marks]

   (c) Use a reduction to prove the undecidability of the VARINEQUALITY problem. VARINEQUALITY is defined as follows. Given a computer program P that takes no input, and two integer variables A and B declared in P, will the value in B ever exceed the value in A during the execution of P? [7 marks]

   (d) Use a reduction to prove the $\mathcal{NP}$-completeness of the HITTINGSET problem. You may assume that HITTINGSET is in $\mathcal{NP}$, and that 3-SAT is $\mathcal{NP}$-complete. HITTINGSET is defined as follows. Given a system $\{A_1, \ldots, A_m\}$ of finite sets and a natural number k, does any set with no more than k elements exist that intersects every A_i? [8 marks]